Neutron stars and the equation of state of dense matter

Tyler Gorda

TU Darmstadt

PhD Retreat, Graz (13-15.05.2022)

TECHNISCHE UNIVERSITÄT DARMSTADT

Lecture 1:
Neutron stars and their observational properties

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Outline

1. What is a Neutron Star (NS)?
2. Basic phenomena in General Relativity
3. Observations of NSs
4. NS structure equations (TOV eqns)

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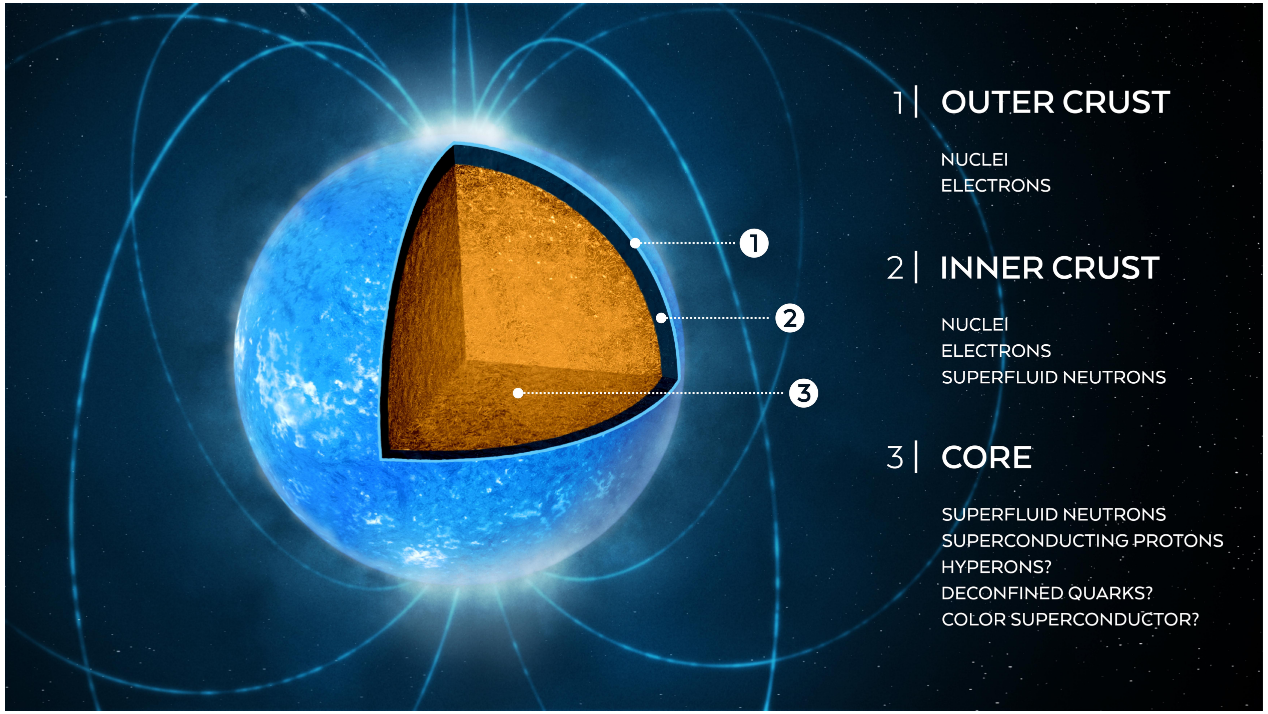
What is a Neutron Star?

NSs are the dense remnants of dead stars, held against gravitational collapse by *repulsive* nuclear/QCD forces



Watts+, Rev. Mod. Phys. 88 (2016)

- Mass $\lesssim 2M_\odot$
- 11 km $\lesssim R \lesssim 13$ km
- $T \lesssim$ keV $\sim 10^7 K$

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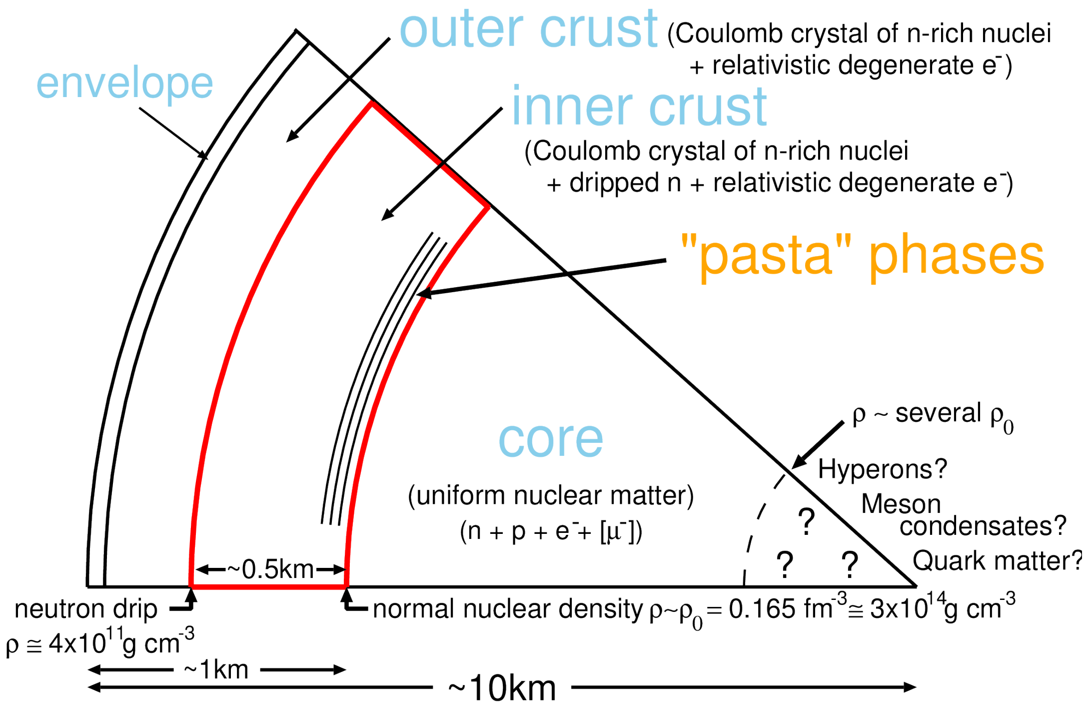

Watanabe and Sonoda, arXiv:cond-mat/0502515

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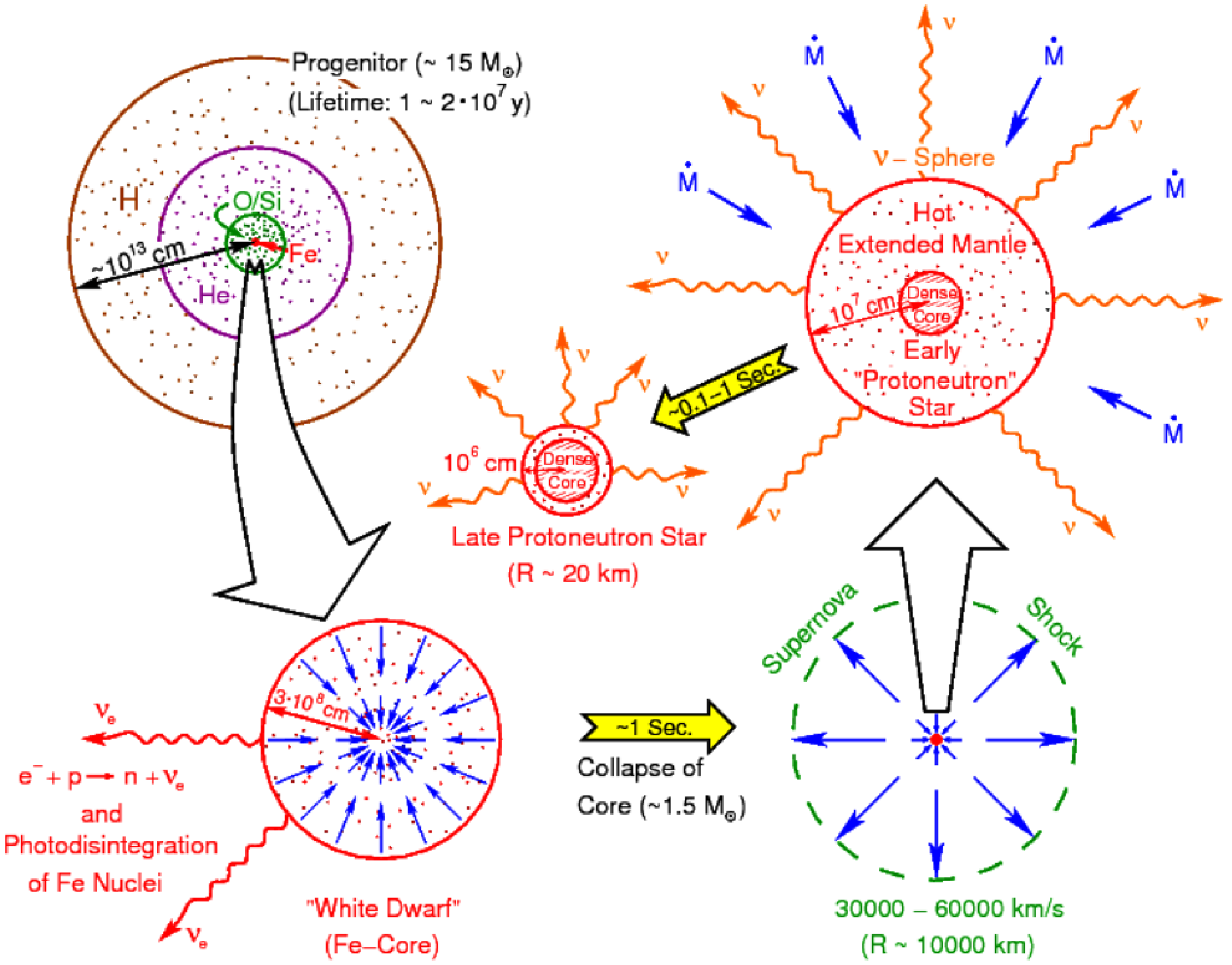
Births and Deaths of NSs: 1

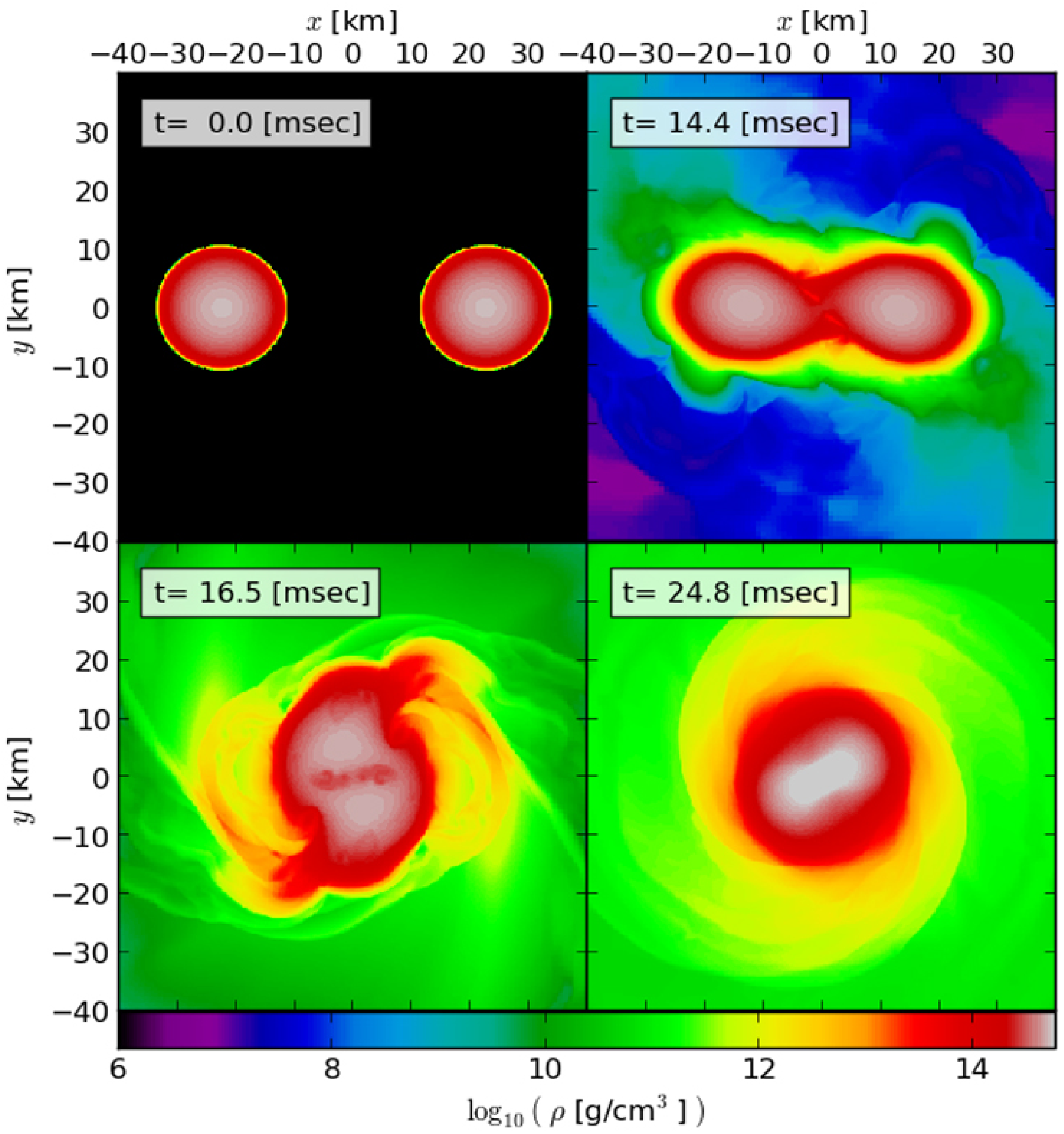
- Produced in *Supernovae*
  - Core collapses, producing massive numbers of neutrinos, forming proto-neutron star
  - Rapidly cools $O(10^2\ \mathrm{s})$ by neutrino emission



Janka, adapted from A. Burrows (1990)

Births and Deaths of NSs: 2

- Some die in *binary NS mergers*
  - Two NSs in tight *inspiral* emit gravitational radiation to spiral closer
  - Eventually, *tidally disrupt*; can eject matter and/or form black hole
  - Can produce Gamma-Ray Burst, and synthesize heavy elements



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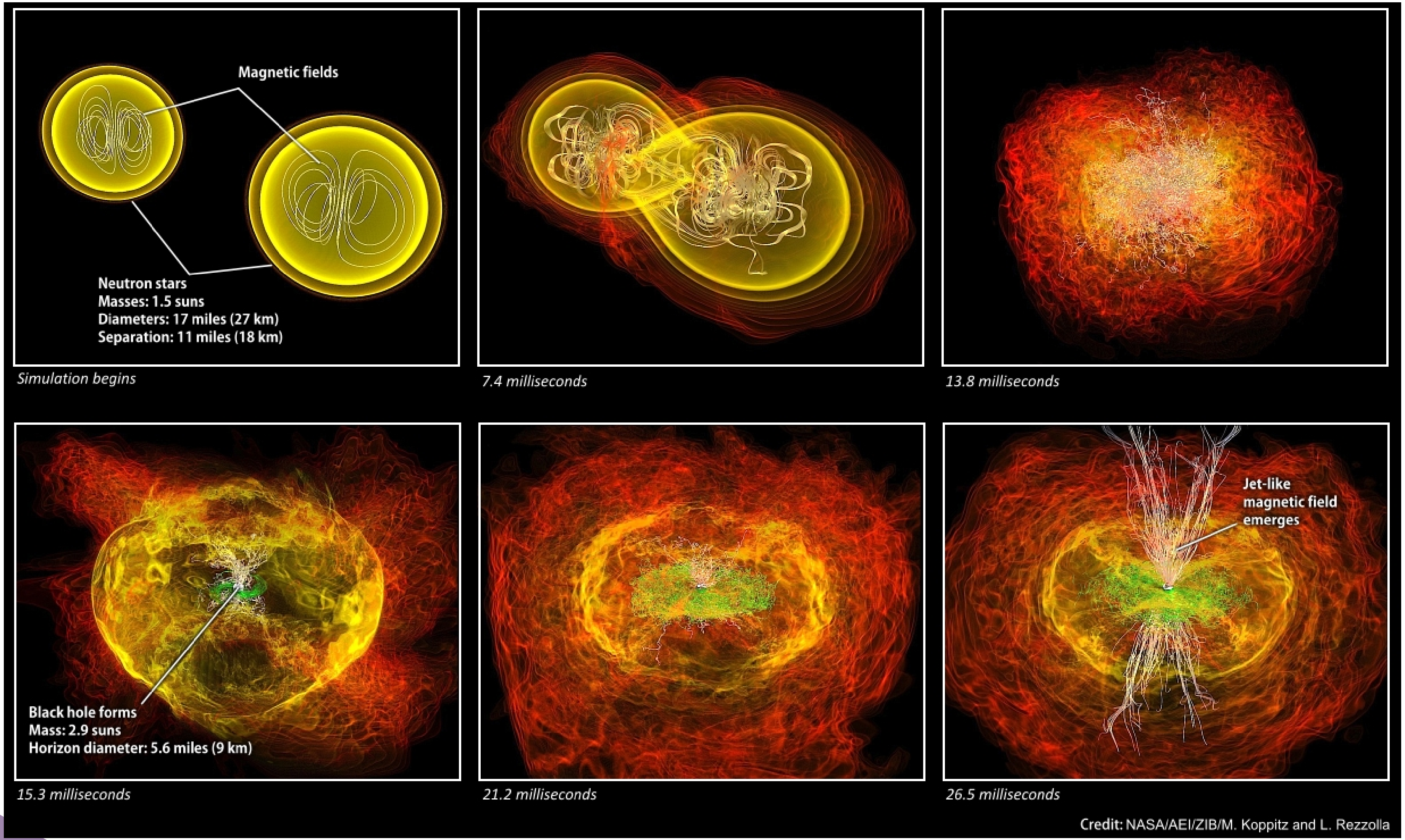
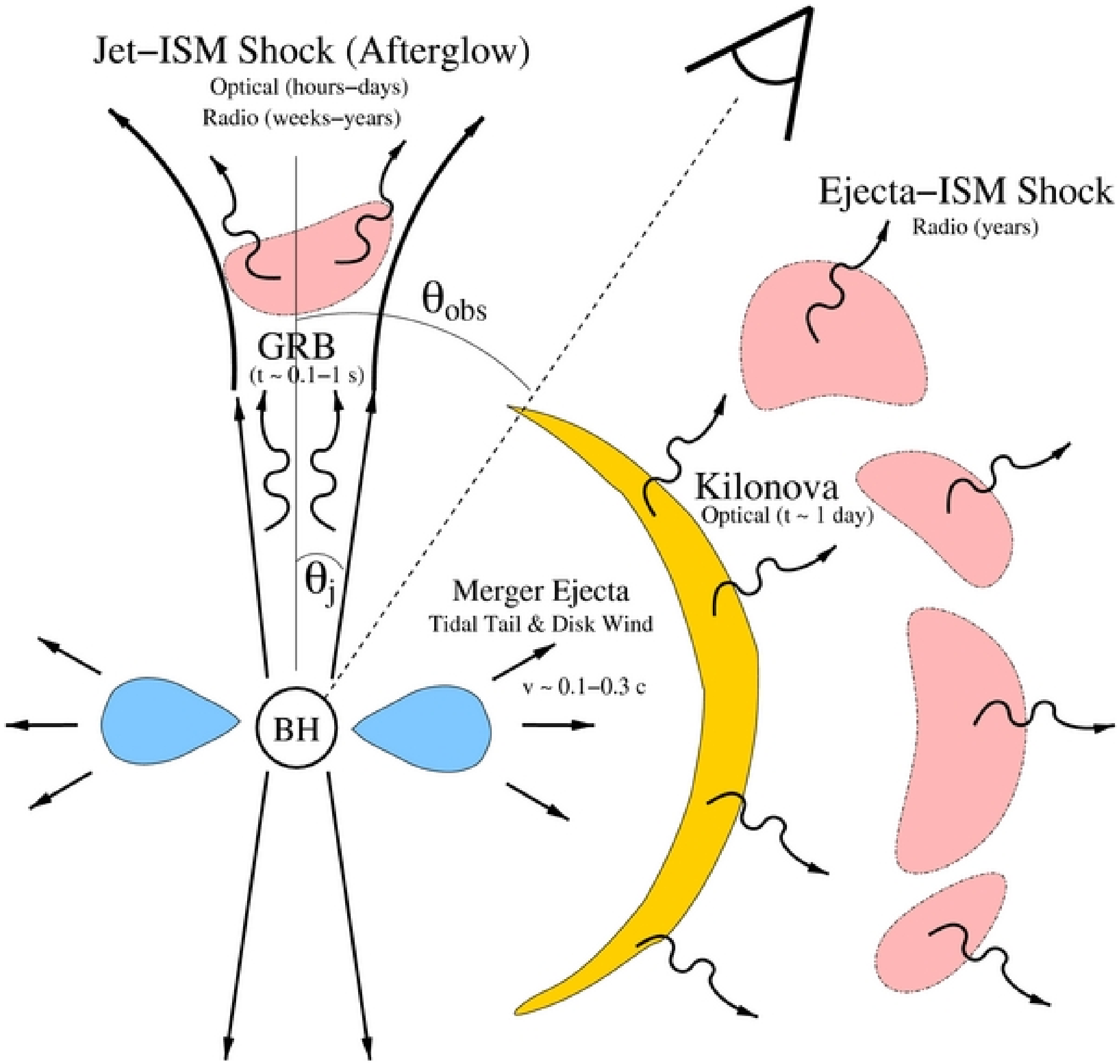


Births and Deaths of NSs: 2

- First 10 ms: *Dynamical ejecta* (originating from the merger)
  - tidal ejecta
  - shock-heated ejecta
- 10 ms – 10 s: *Post-merger ejecta* (originating from the accretion disk)
  - neutrino-driven winds
  - viscous ejecta (turbulence)
- Days: *Kilonova*
- Up to 100s of days: *Afterglow* of a Gamma-ray burst
  - related to relativistic jets



Metzger & Berger, ApJ 746 (2012)

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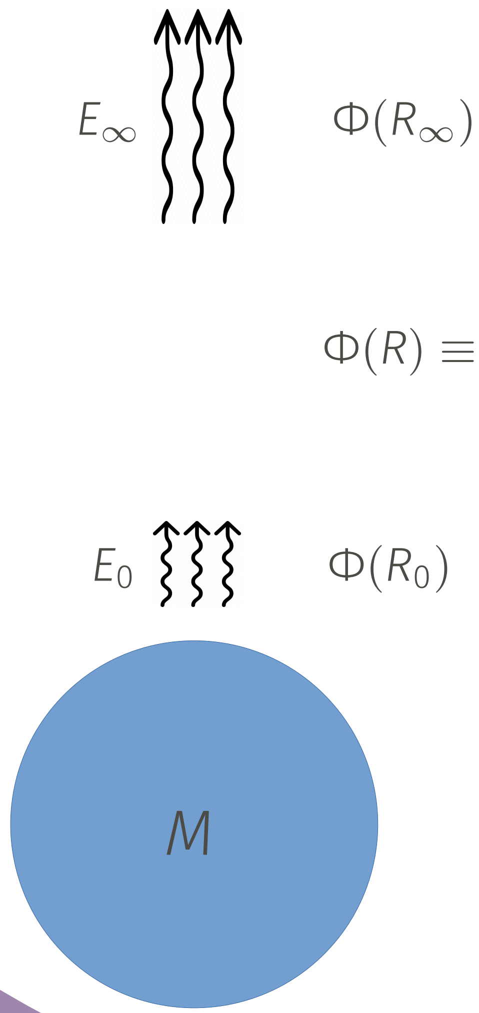
GR Phenomena 1: Gravitational time dilation/Redshift



$$\Phi(R) \equiv -\frac{GM}{R}$$

Photons climbing out of Gravitational Well experience *redshift*.

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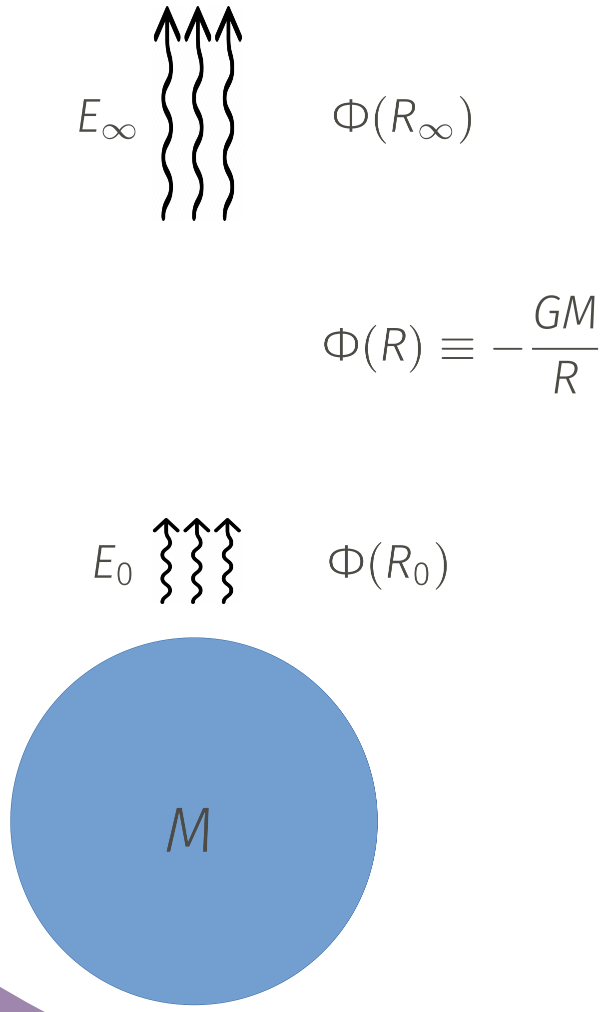

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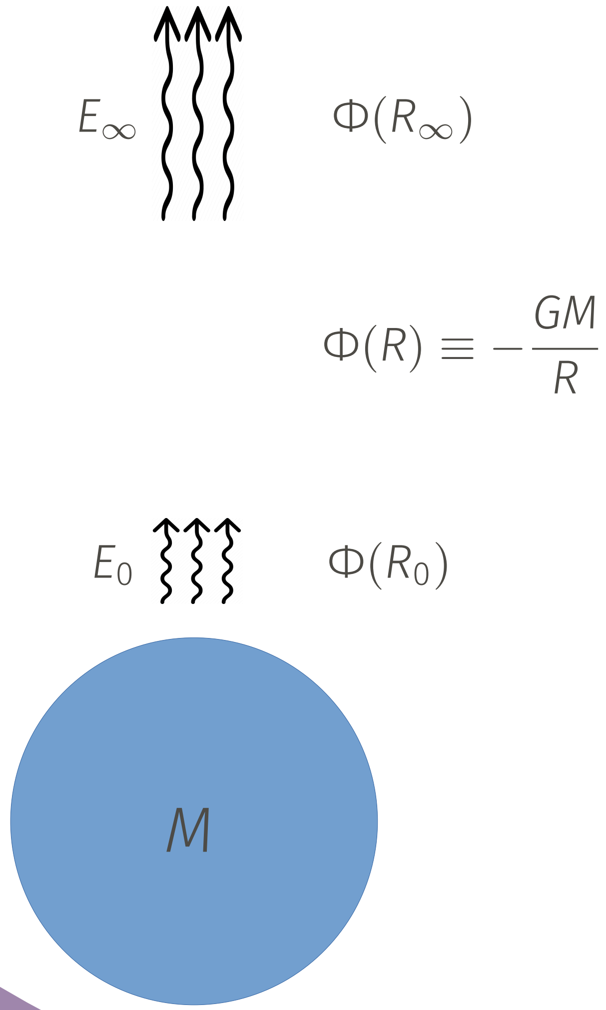

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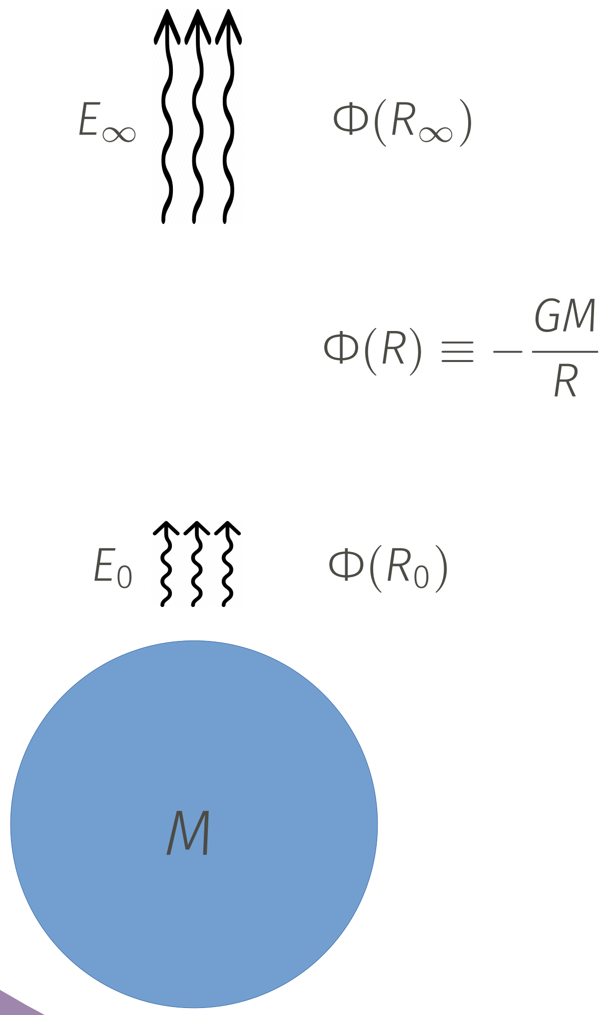

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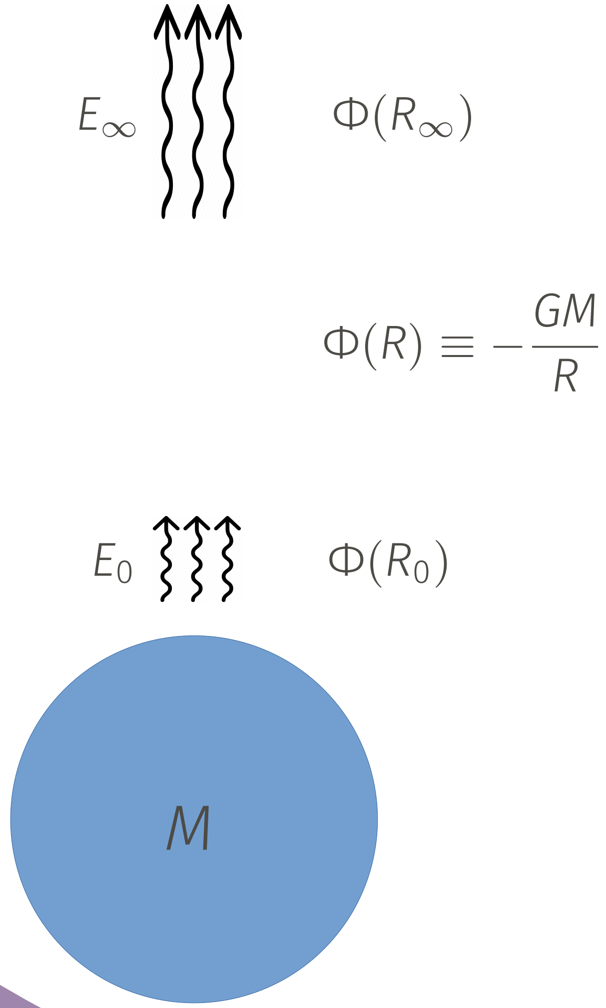

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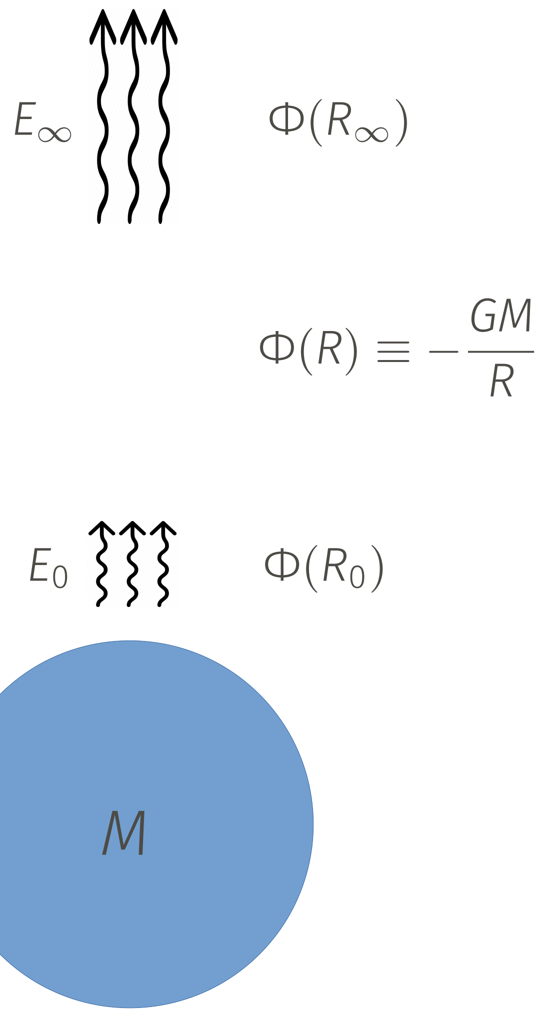

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$$\left(\text{Actual correct formula for point mass is } \frac{E_0}{E_\infty} = \frac{\sqrt{1 + 2\Phi(R_\infty)/c^2}}{\sqrt{1 + 2\Phi(R_0)/c^2}}\right)$$

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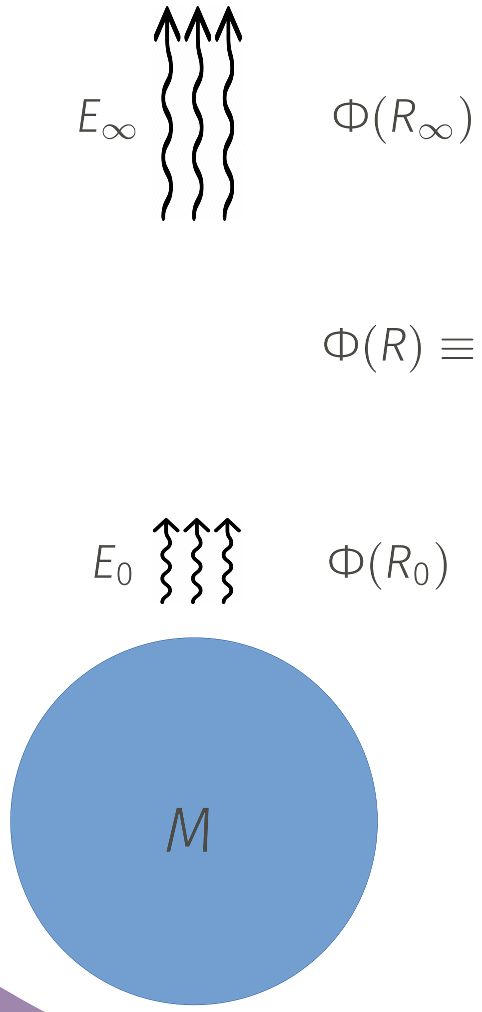

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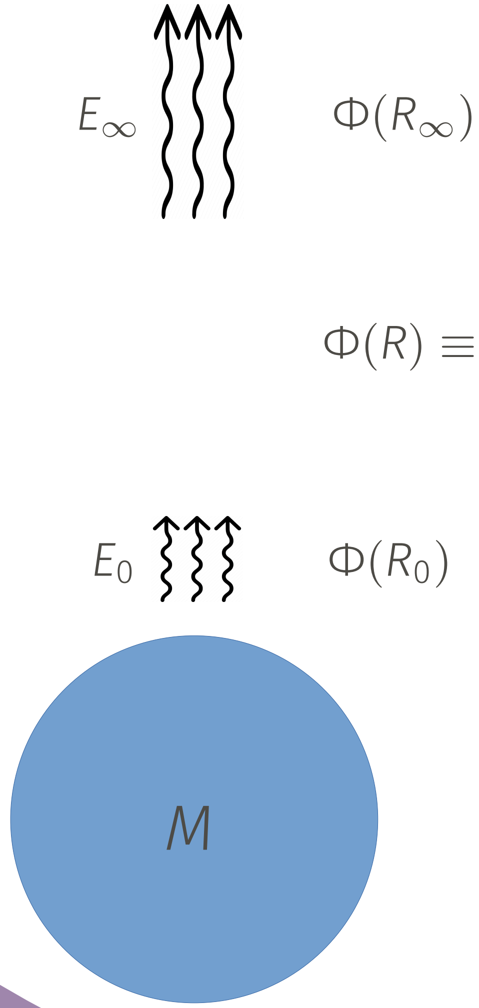

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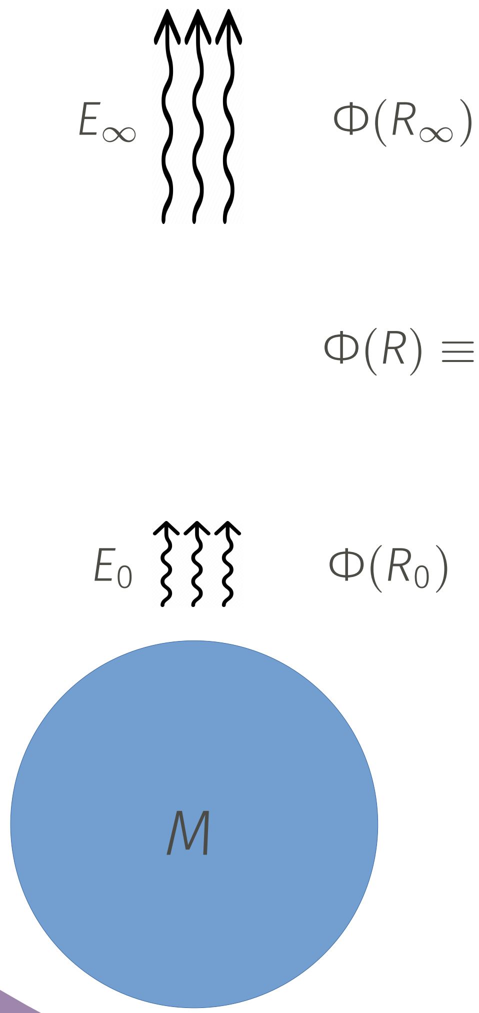

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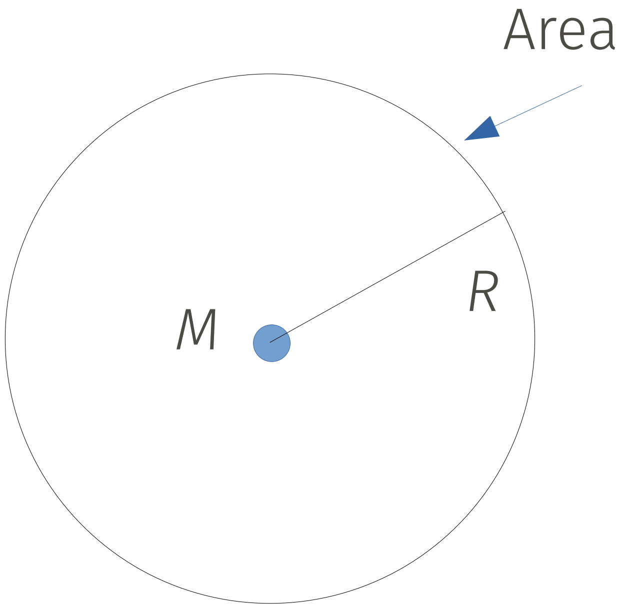
Time interval at coordinate *R*

GR Phenomena 2: Space curving



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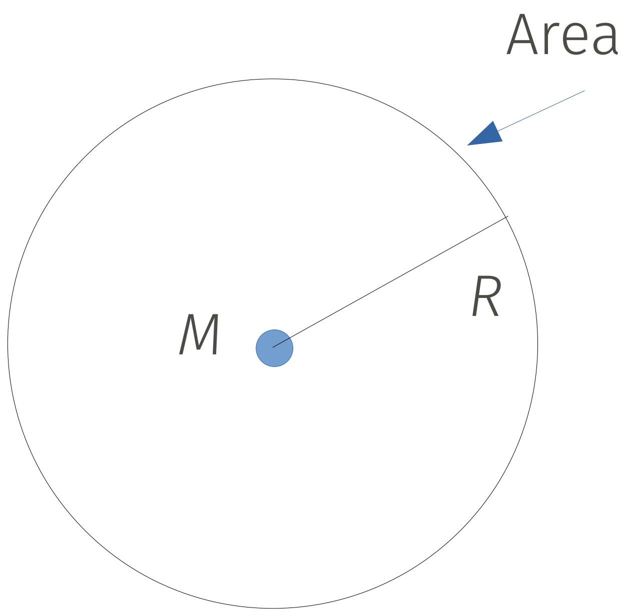

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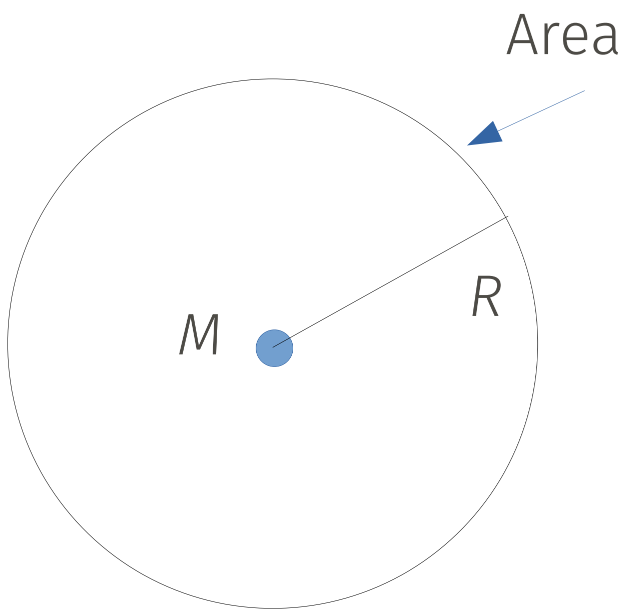

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Area ***increases slower*** with distance than expected

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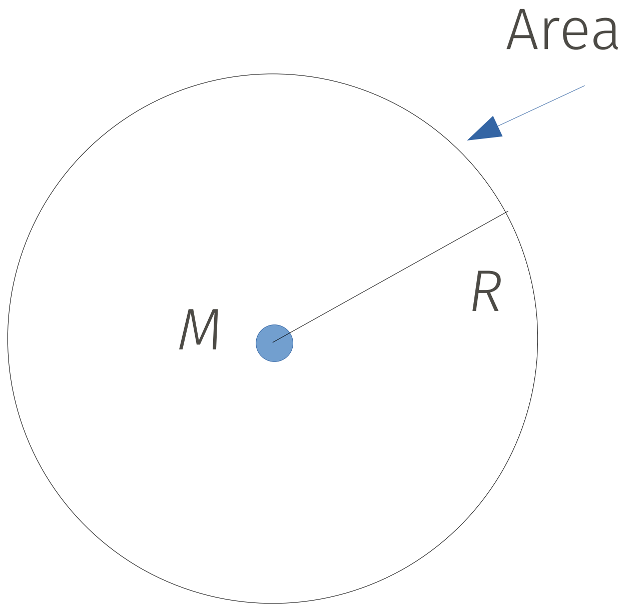

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Often, one defines ***areal radius*** r such that Area = $4\pi r^2$, but $r \neq R$. Then the spatial line element is

$$ds^2 = \frac{dr^2}{[1 + 2\Phi(r)/c^2]} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

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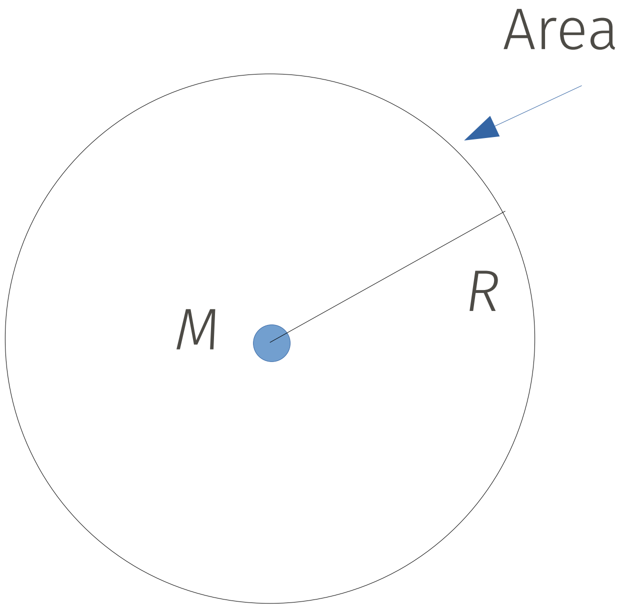

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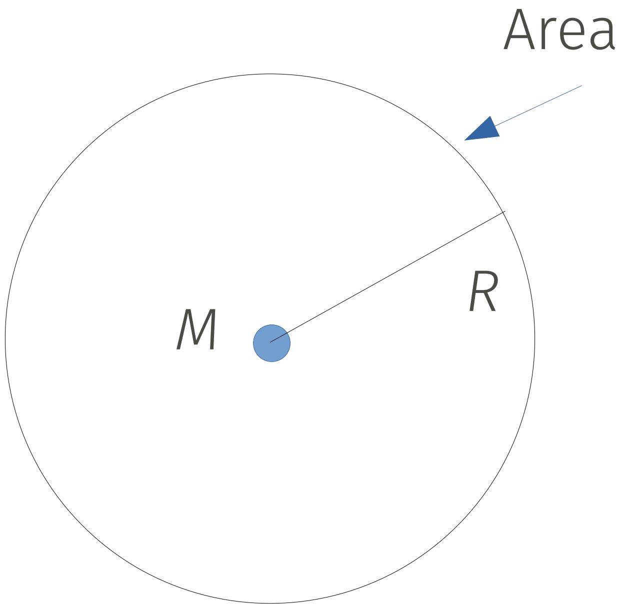

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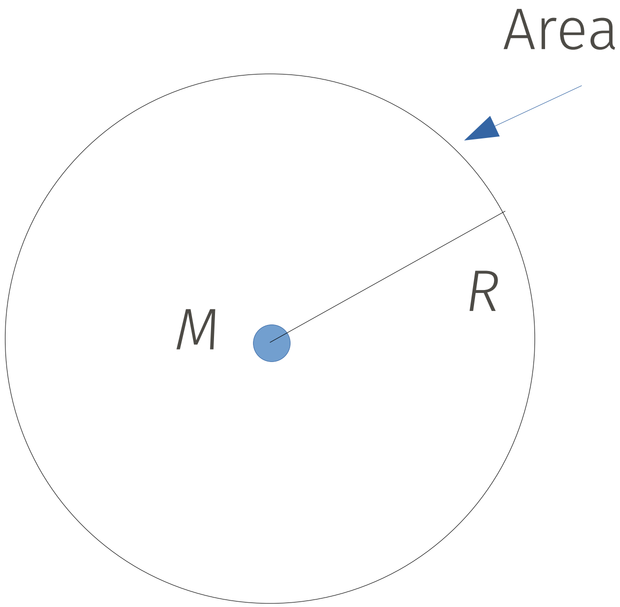

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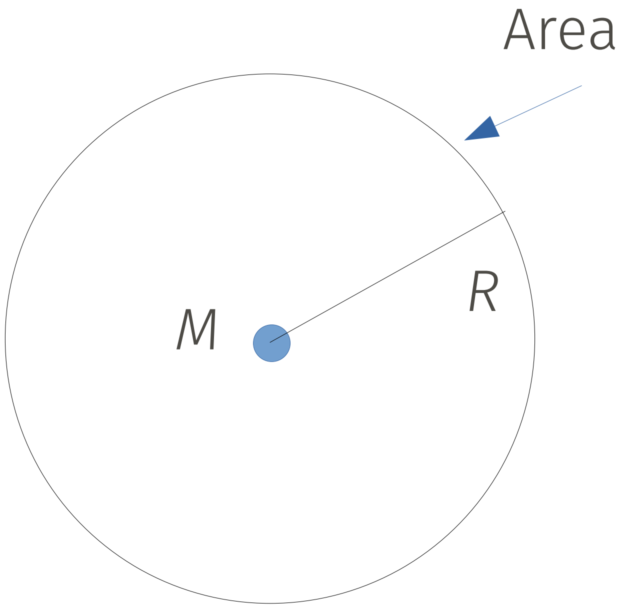

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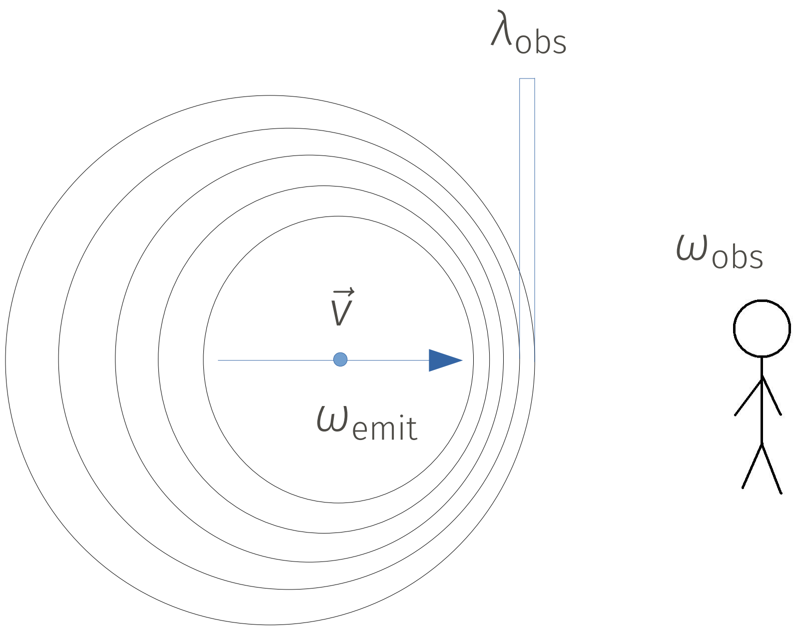
*Also leads to gravitational lensing

GR Phenomena 3: Doppler shift



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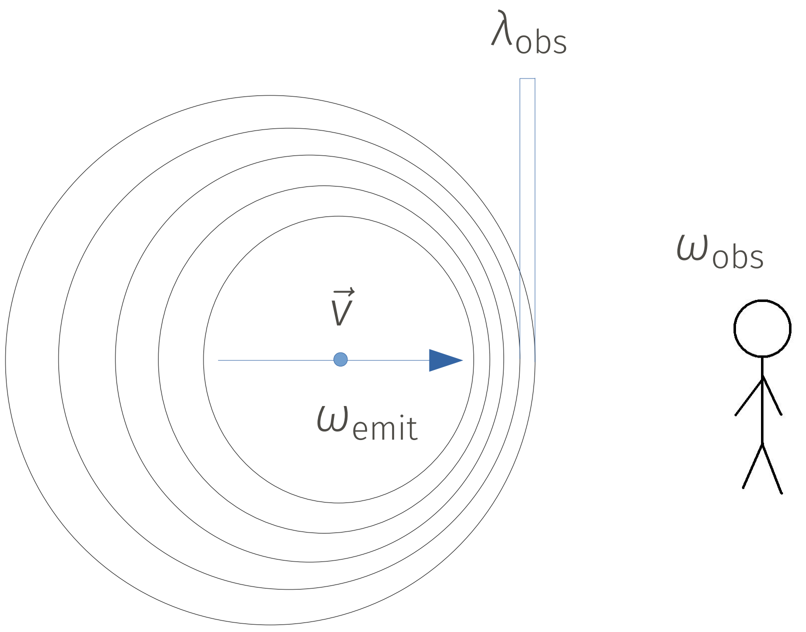

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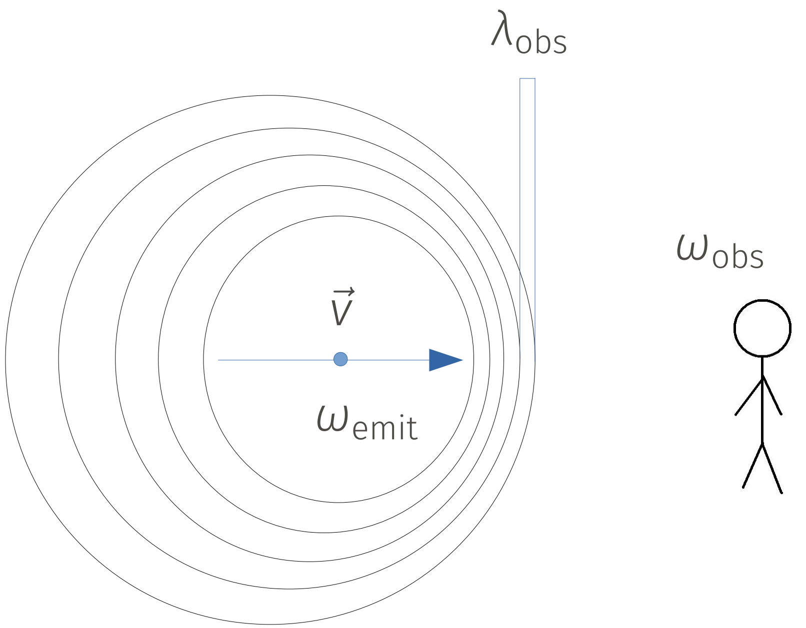

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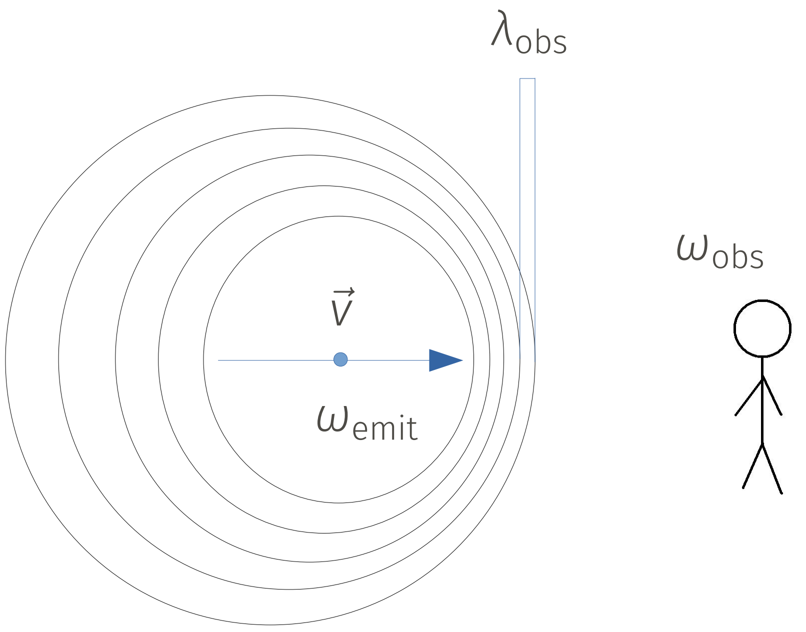

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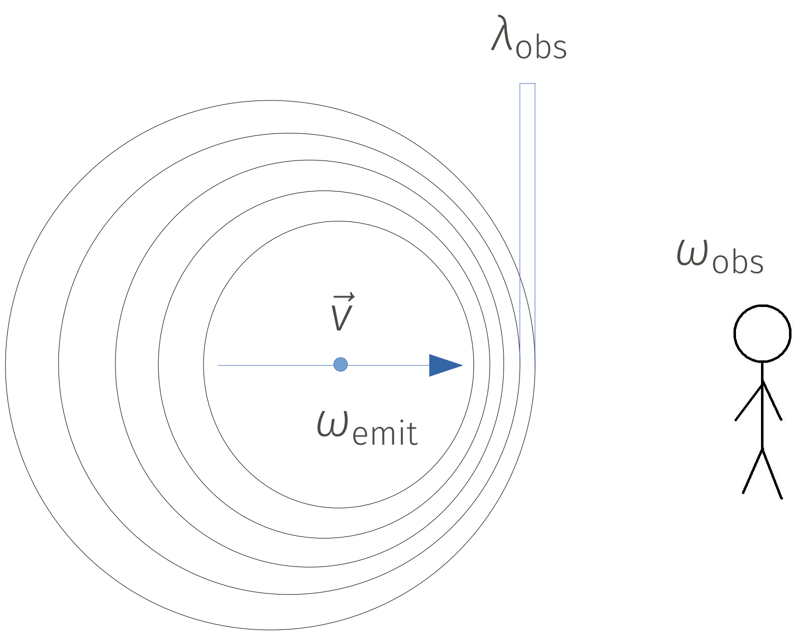

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(Correct relativistic expression)

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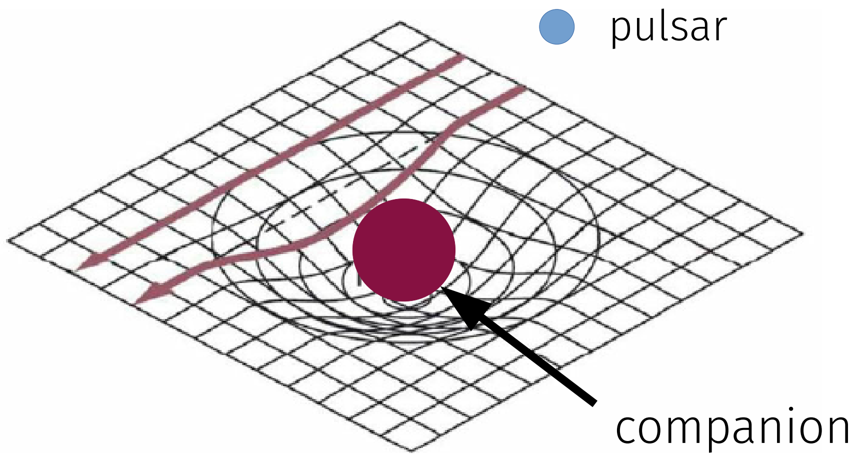
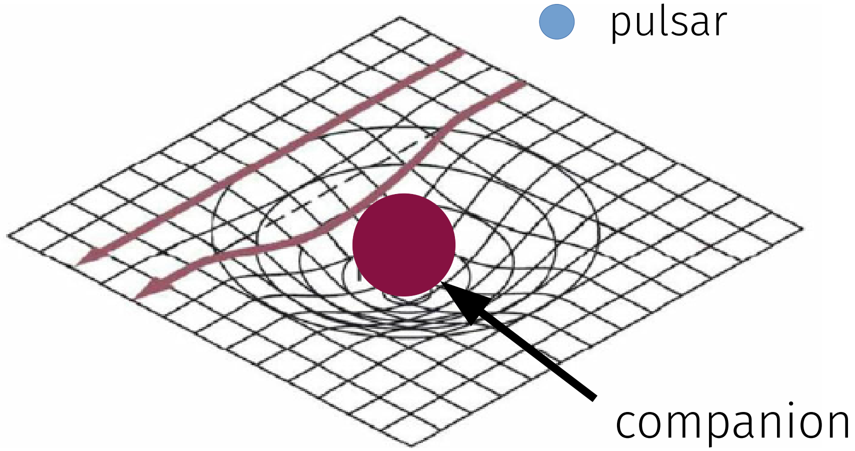


Heinkelmann & Schuh, Proc. Int. Astron. Union, 261 (2010).

Measuring NS masses

- *Shapiro delay* of pulsar signals in eclipsing, edge-on binaries
- Pulses delayed in GR, since the space-time is warped
- Extract orbital parameters from delay times
- $$M_{\mathrm{Max}} \geq \begin{cases} 1.97 \pm 0.04 M_\odot & \text{PSR J1614-2230} \\ 2.01 \pm 0.04 M_\odot & \text{PSR J0348+0432} \\ 2.08 \pm 0.07 M_\odot & \text{PSR J0740+6620} \end{cases}$$

Demorest+ Nature 467 (2010),
Antoniadis+ Science 240 (2013),
Fonseca+ 2104.00880



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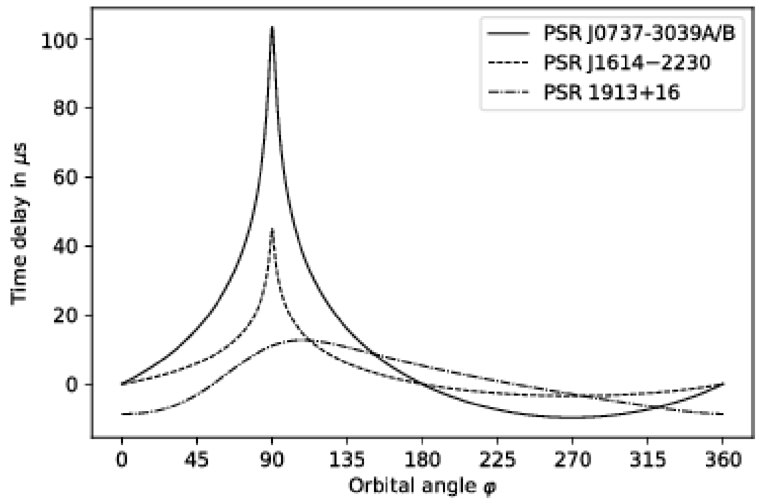

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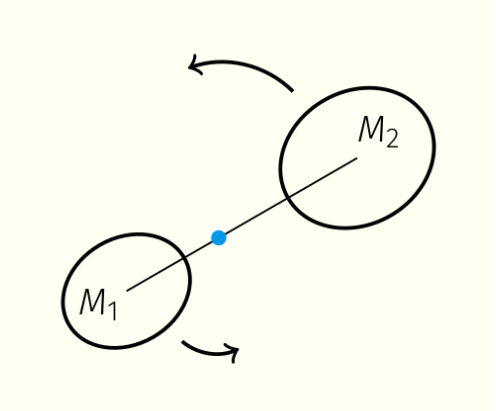
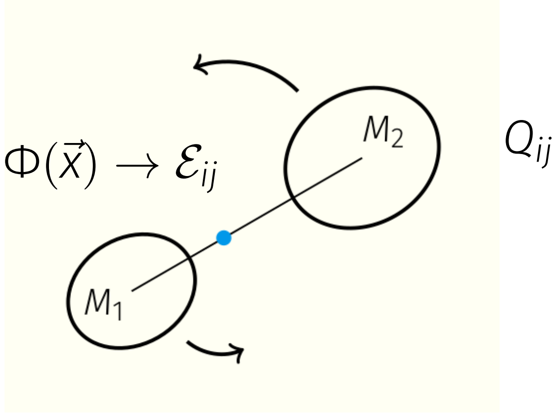


Measuring NS deformabilities

- *Inspiral phase* binary-NS merger sensitive to deformability of stars:

$$\Lambda(M) \equiv |Q_{ij}/\mathcal{E}_{ij}|M^5$$

- Less pointlike → more deformed → Radiate more GWs → merge sooner



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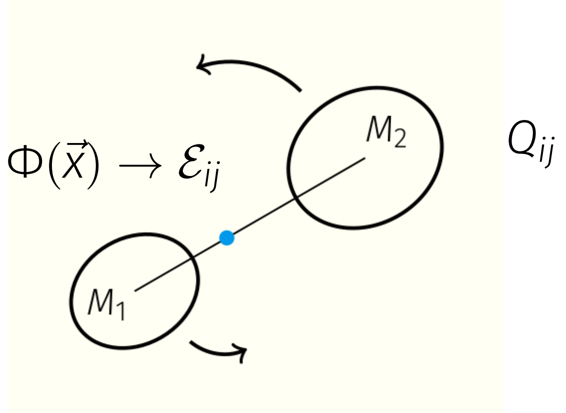
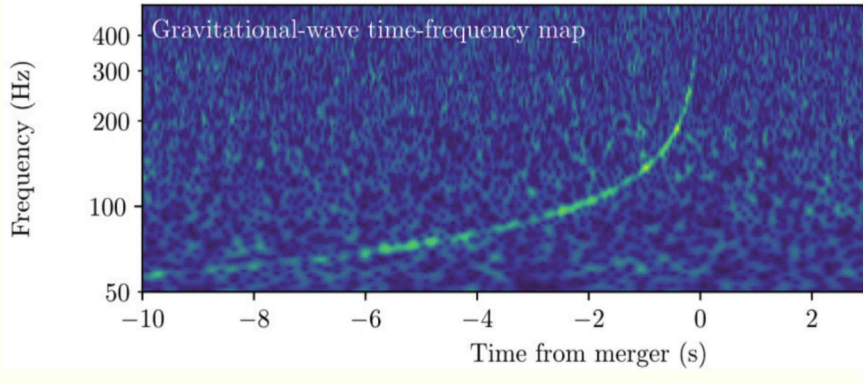
- $$\boxed{\begin{aligned}&\tilde{\Lambda} < 720,\ \text{with } \mathcal{M}_{\mathrm{chirp}} = 1.186 M_\odot, \\ &q \equiv M_2/M_1 \in [0.7, 1] \qquad \text{GW170817}\end{aligned}}$$

Abbott+ Phys. Rev. Lett. 119 (2017); Phys. Rev. Lett. 121 (2018); Phys. Rev. X 9 (2019).

$$\tilde{\Lambda} \equiv \frac{16}{13}\left[\frac{(M_1 + 12M_2)M_1^4}{(M_1+M_2)^5}\Lambda(M_1) + (1 \leftrightarrow 2)\right];$$

$$\mathcal{M}_{\mathrm{chirp}} \equiv \frac{(M_1 M_2)^{3/5}}{(M_1+M_2)^{1/5}}$$





Measuring NS deformabilities

- *Inspiral phase* binary-NS merger sensitive to deformability of stars:
$$\Lambda(M) \equiv |Q_{ij}/\mathcal{E}_{ij}|M^5$$
- Less pointlike → more deformed → Radiate more GWs → merge sooner
- $\tilde{\Lambda} < 720, \text{ with } \mathcal{M}_{\rm chirp} = 1.186 M_\odot,$
$q \equiv M_2/M_1 \in [0.7, 1]$ GW170817

Abbott+ Phys. Rev. Lett. 119 (2017); Phys. Rev. Lett. 121 (2018); Phys. Rev. X 9 (2019).

$$\tilde{\Lambda} \equiv \frac{16}{13}\left[\frac{(M_1 + 12M_2)M_1^4}{(M_1+M_2)^5}\Lambda(M_1) + (1 \leftrightarrow 2)\right];$$

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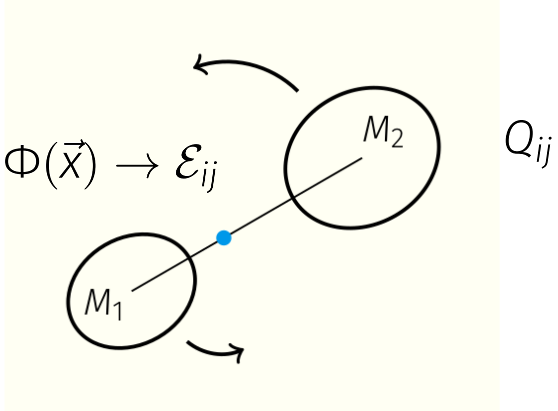
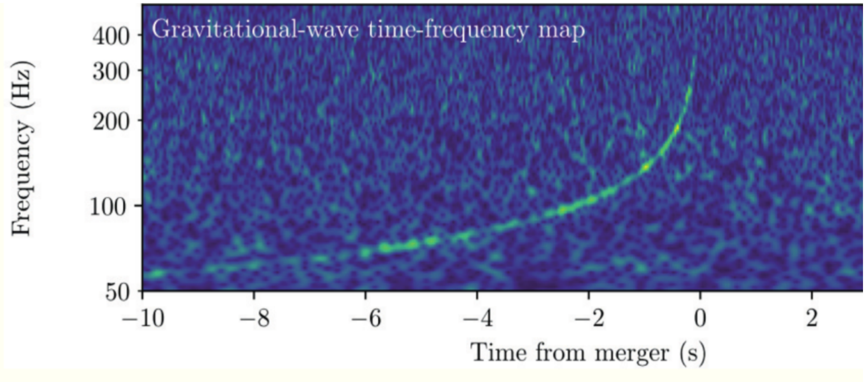




* EM counterpart evidence for collapse to BH (BH-hyp)

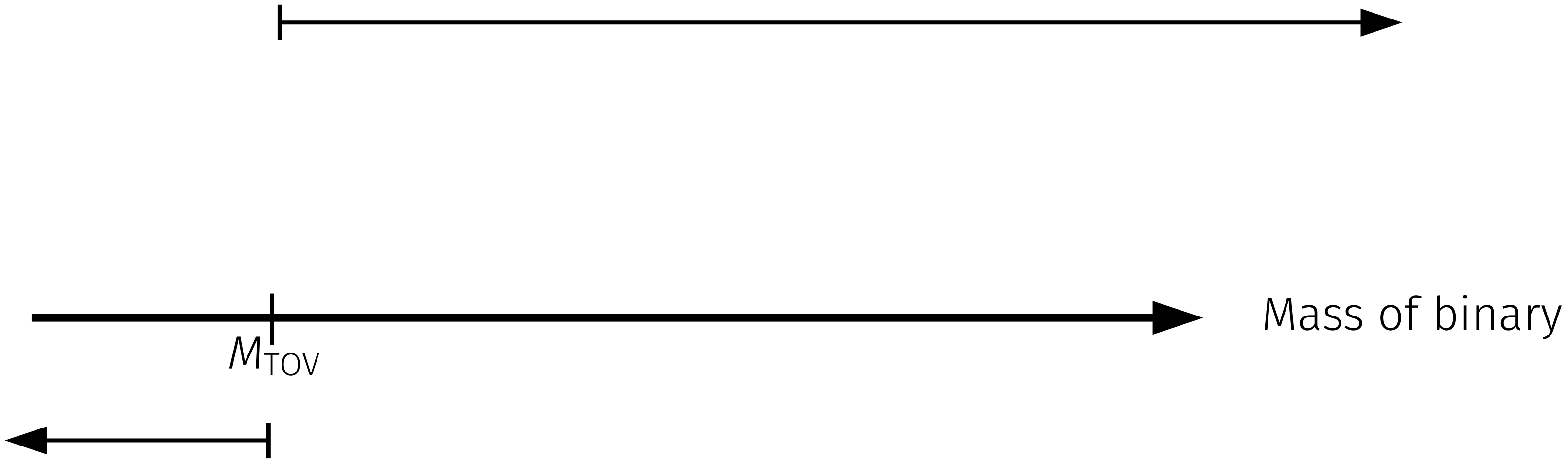
Margalit & Metzger, Astrophys. J. Lett. 850, (2017); Rezzolla+ Astrophys. J. Lett. 852, (2018); Ruiz+ Phys. Rev. D 97, (2018)

Possible binary mergers (GW170817)

Mass of binary

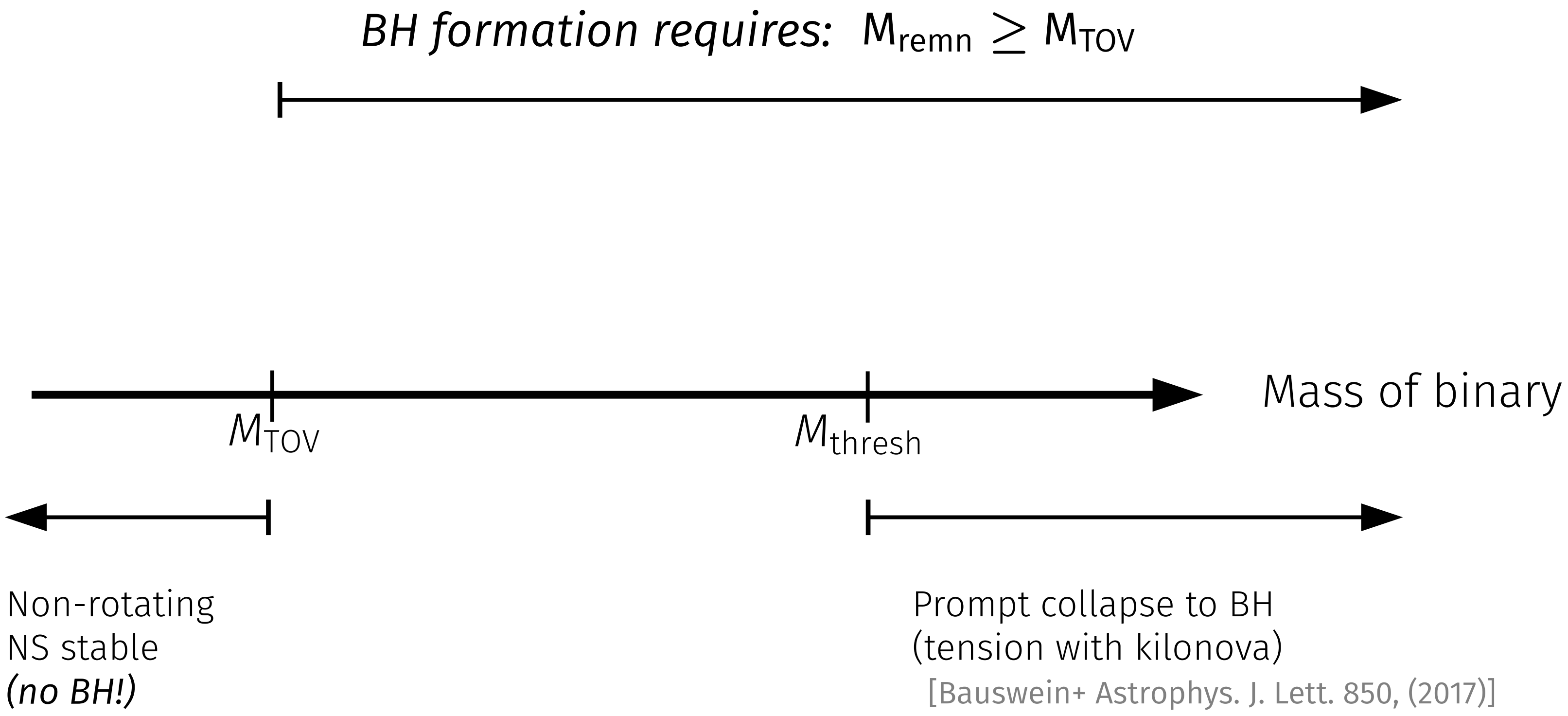
Possible binary mergers (GW170817)

BH formation requires: $M_{\mathrm{remn}} \geq M_{\mathrm{TOV}}$



Non-rotating
NS stable
(no BH!)

Possible binary mergers (GW170817)



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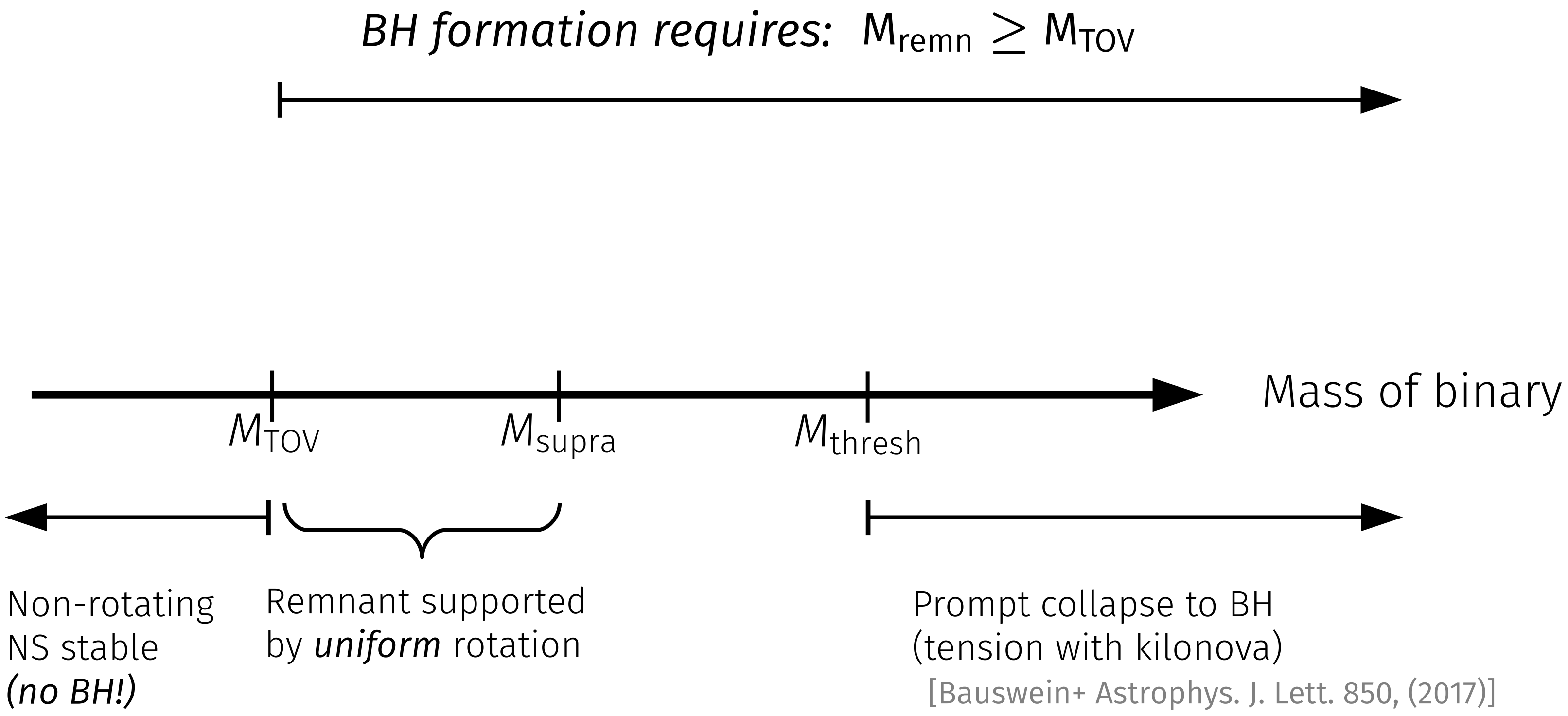

[Bauswein+ Astrophys. J. Lett. 850, (2017)]

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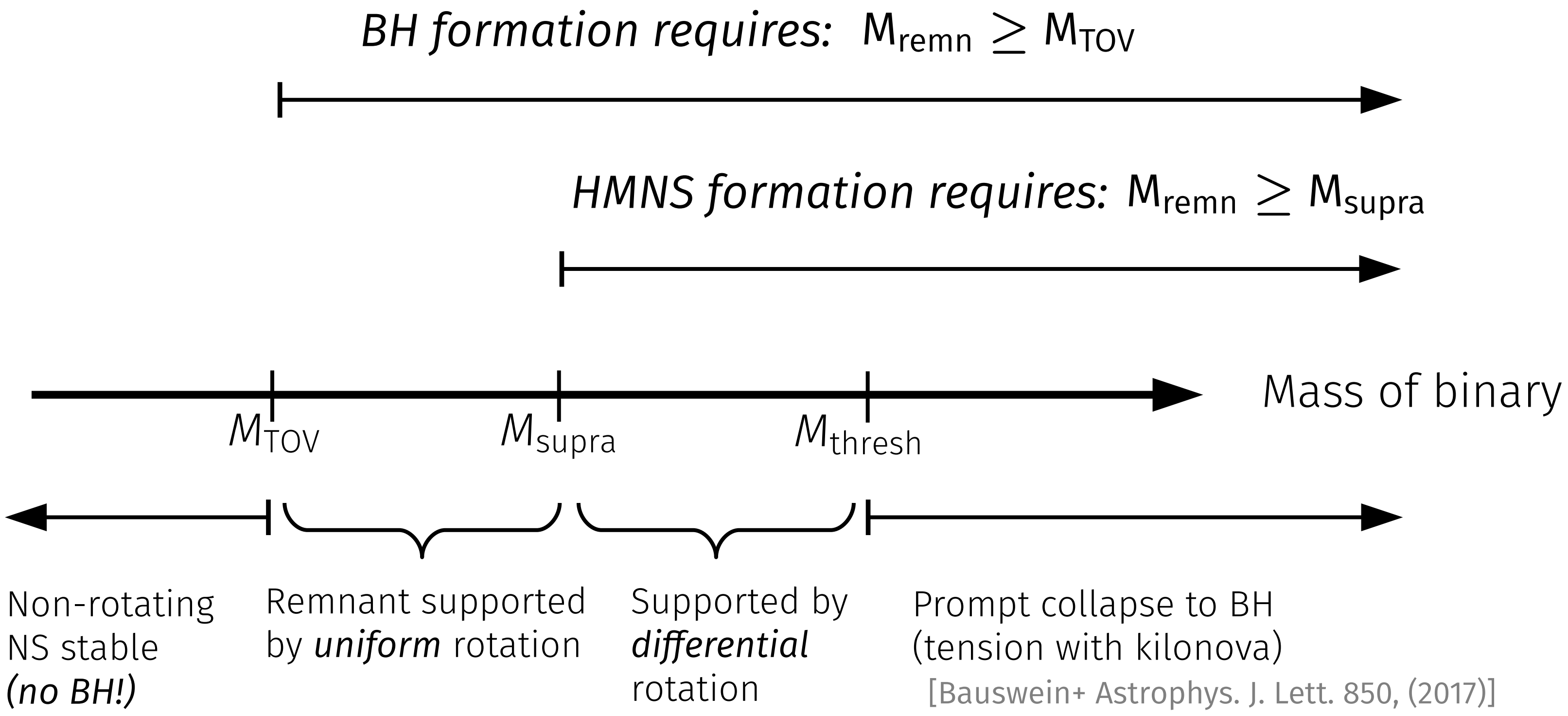

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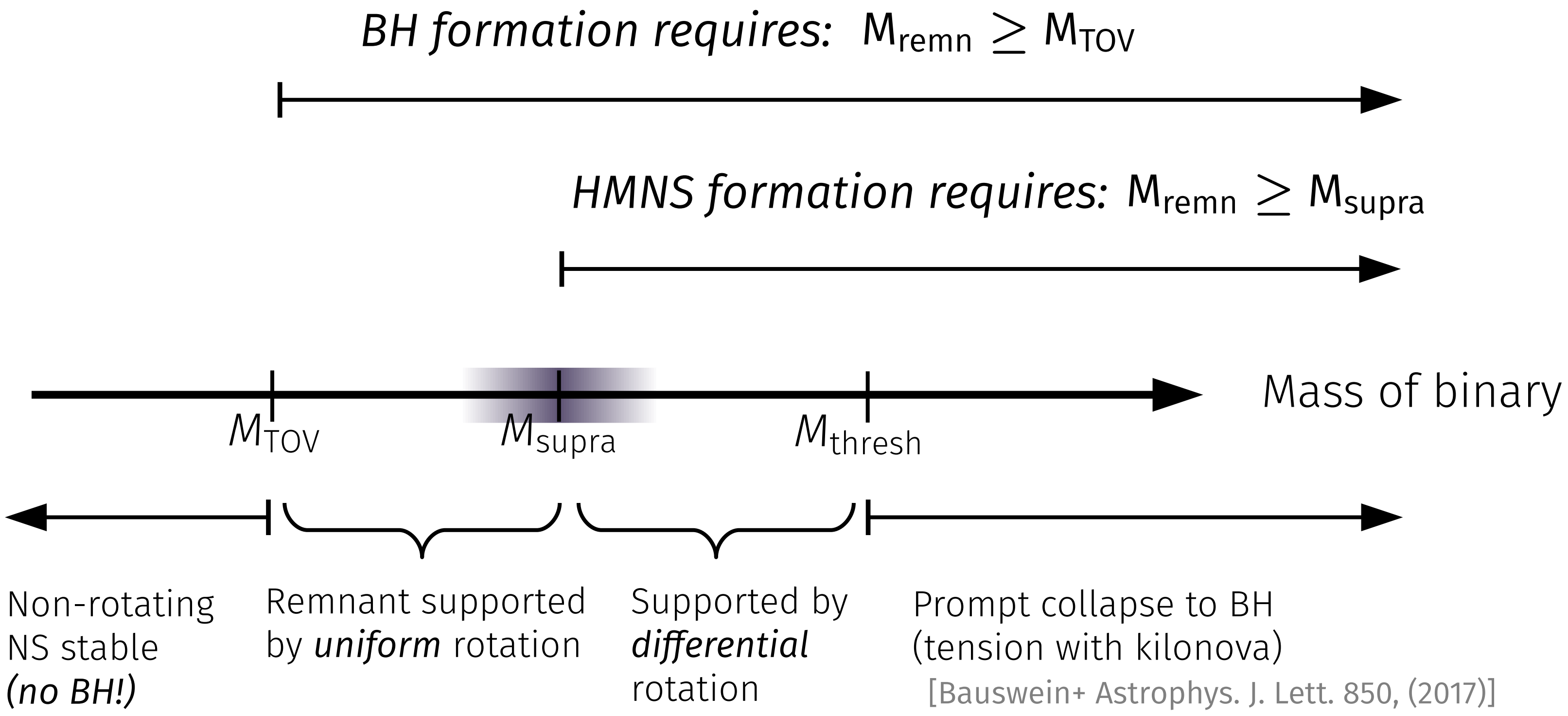
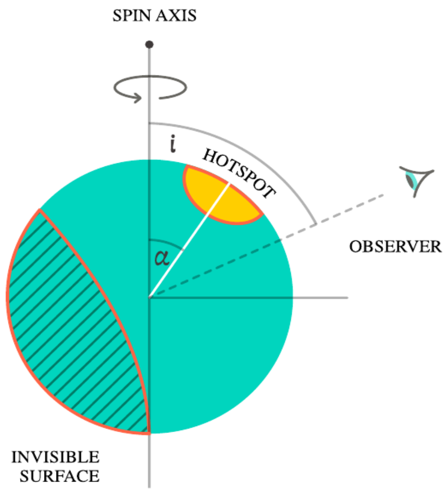


*kilonova and GRB suggest BH formed near M_{supra}

Margalit and Metzger, Astrophys. J. Lett. 850, (2017);
Rezzolla+ Astrophys. J. Lett. 852, (2018);
Ruiz+ Phys. Rev. D 97, (2018)

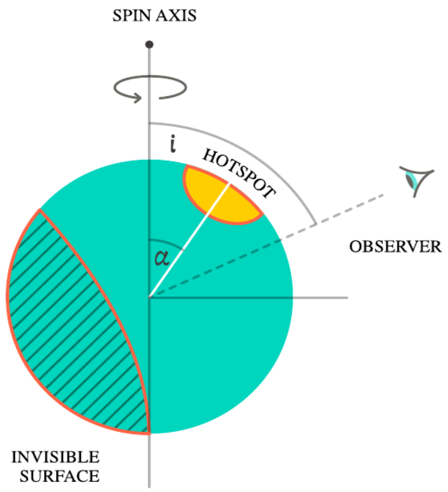
What can observations tell us?

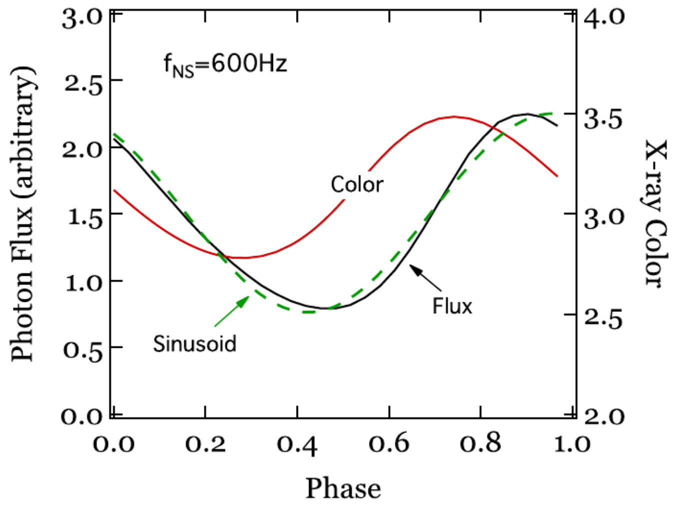
- Masses
- Deformabilities
- **Radii**



Measuring NS radii

- Hotspot on (rapidly) rotating NS generates modulated "pulses" – flux, and X-ray energy (from redshifting)
- *Pulse profile modeling* of hotspot emission sensitive to M/R, or R
- M and R imprinted on pulse profiles → disentangle using ***pulse profile modeling***





Watts+, Rev. Mod. Phys. 88 (2016)

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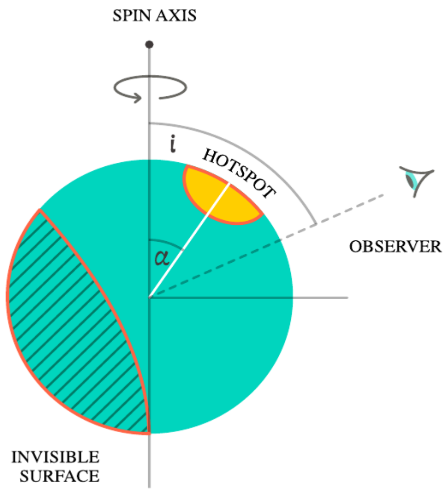
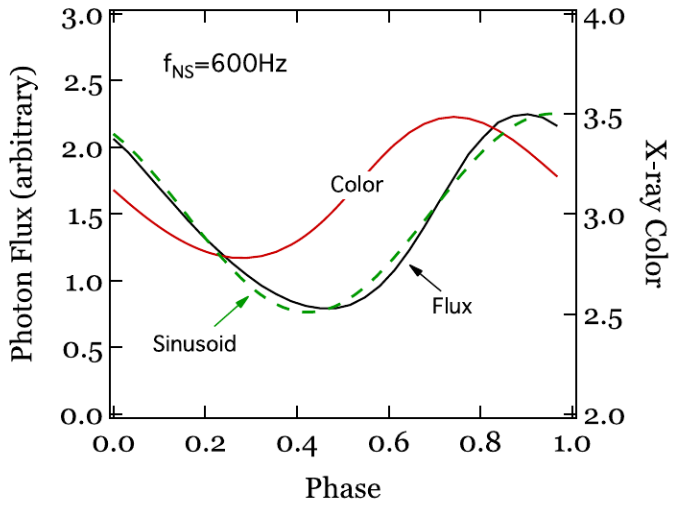
- *Pulse profile modeling* of hotspot emission sensitive to M/R, or R
- M and R imprinted on pulse profiles → disentangle using ***pulse profile modeling***
- $\boxed{R(2M_\odot) \geq 11.0 \text{ km} \quad \text{PSR J0740+6620}}$

Riley+, Astrophys. J. Lett 918 (2021), Miller+ Astrophys. J. Lett. 918 (2021) (NICER)



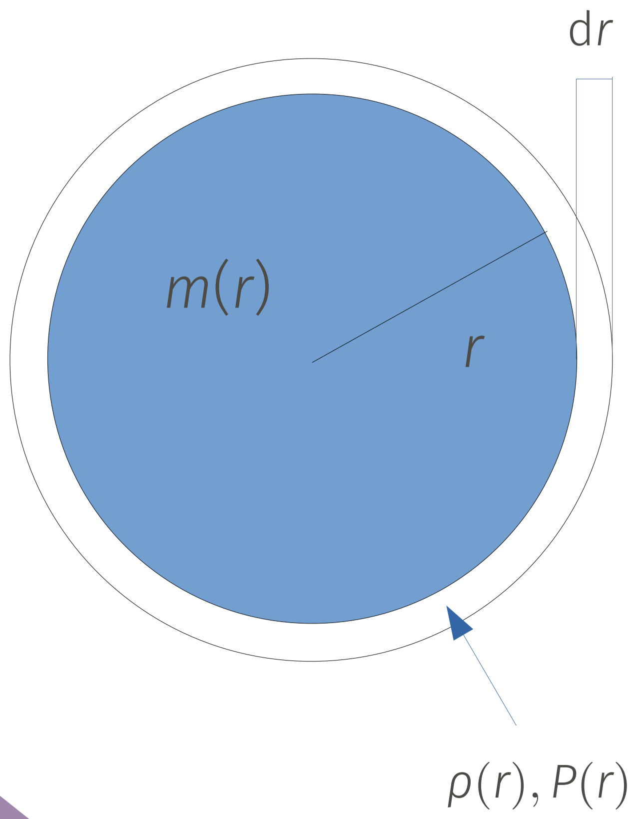


Watts+, Rev. Mod. Phys. 88 (2016)

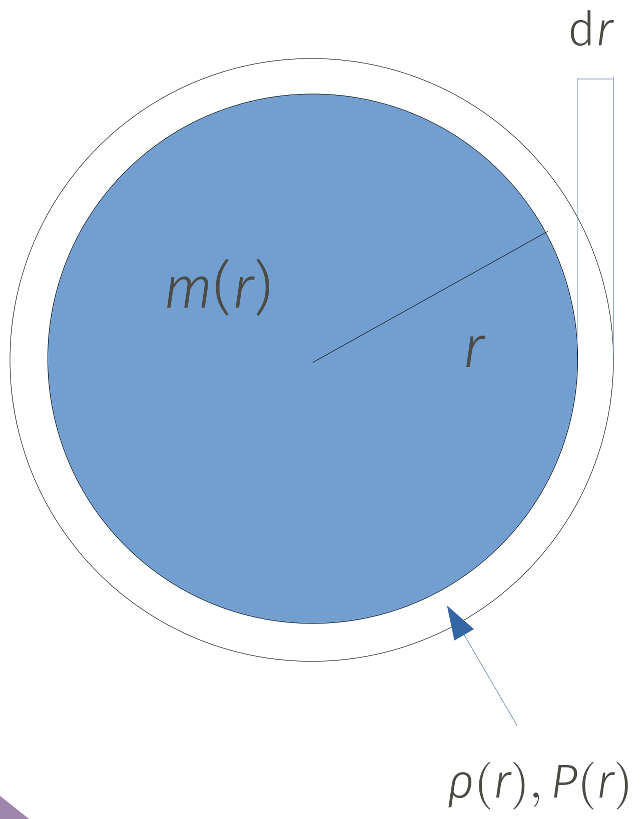
Outline

1. What is a Neutron Star (NS)?
2. Basic phenomena in General Relativity
3. Observations of NSs
4. **NS structure equations (TOV eqns)**

Tolman-Oppenheimer-Volkoff (TOV) equation



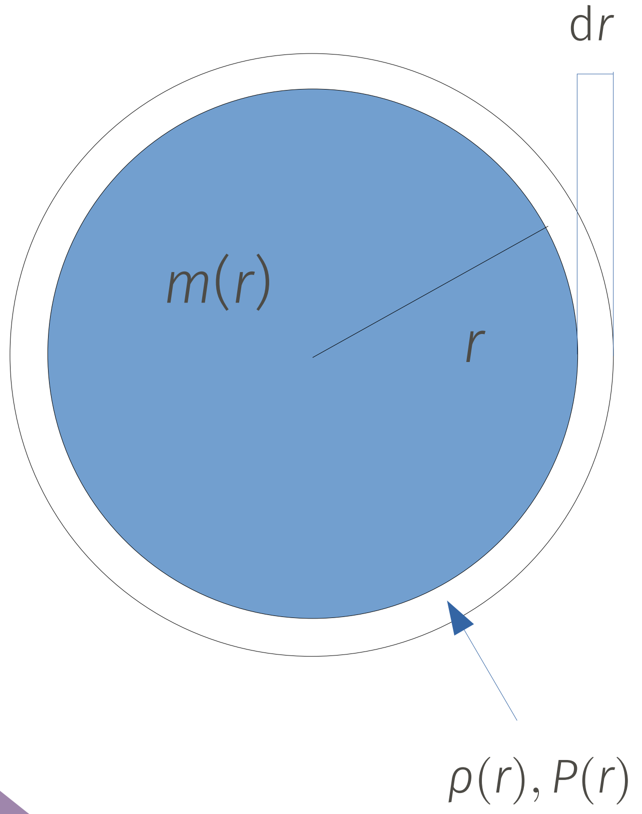
Tolman-Oppenheimer-Volkoff (TOV) equation



In shell of width dr, the following mass is enclosed:

$$\mathrm{d}m = 4\pi r^2 \rho(r)\mathrm{d}r$$

Tolman-Oppenheimer-Volkoff (TOV) equation



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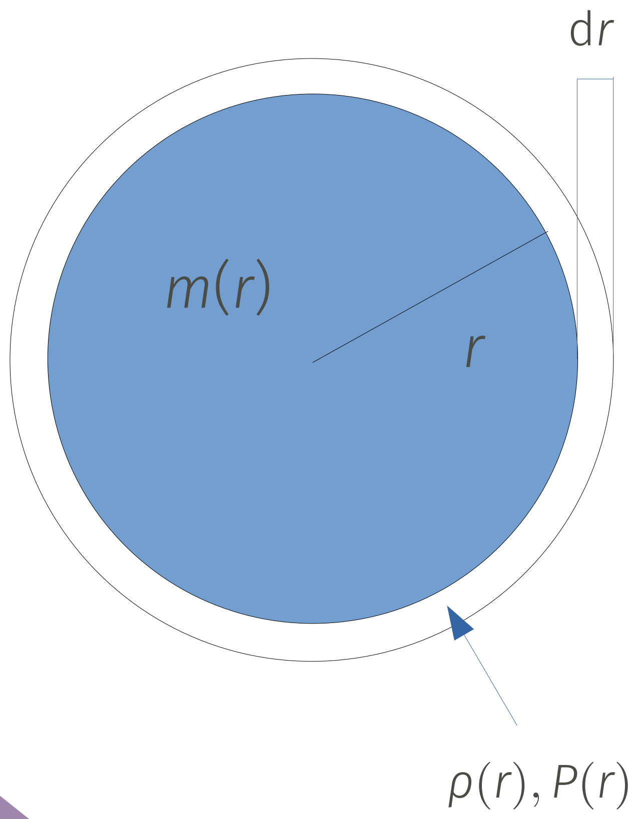

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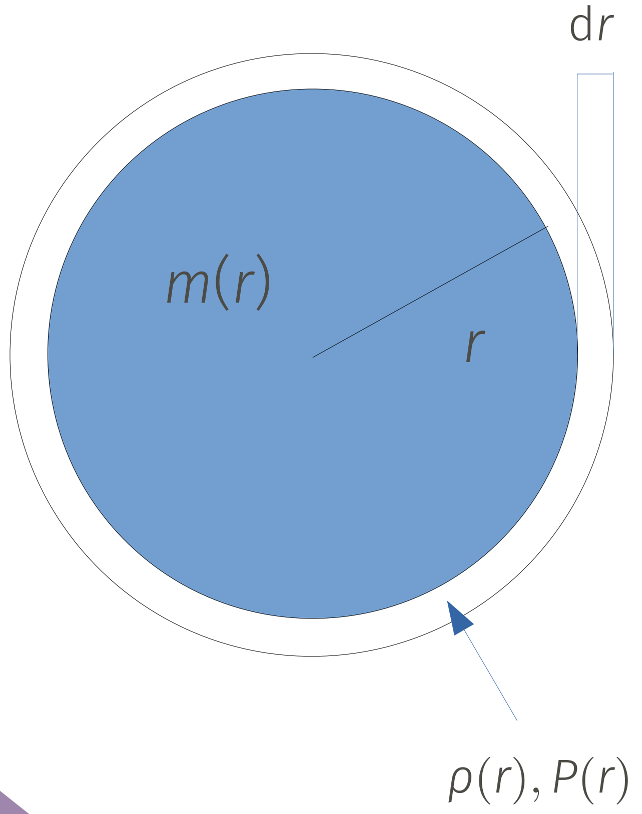

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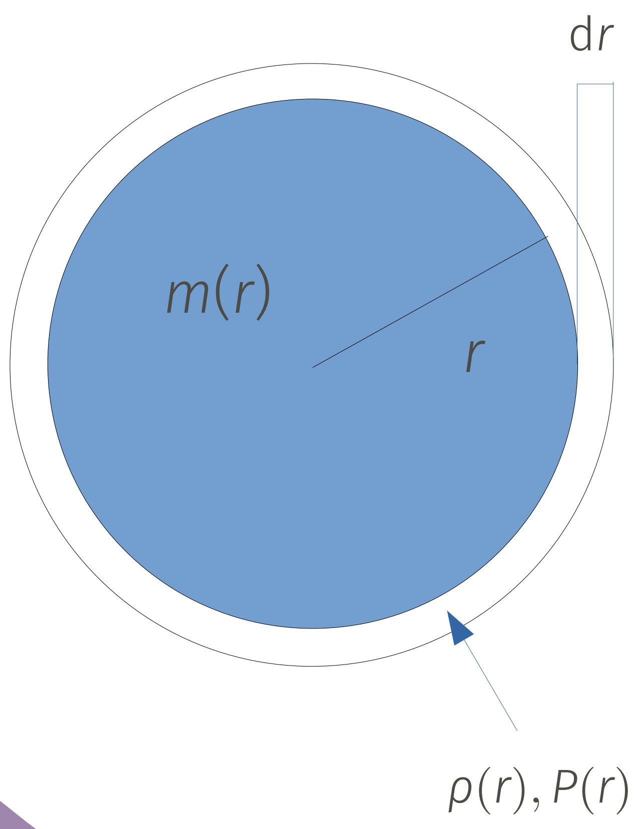

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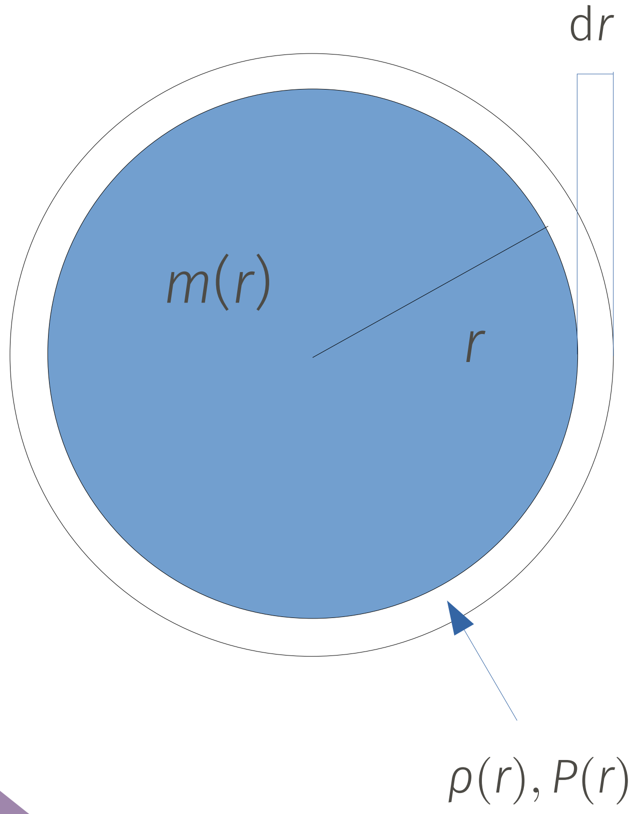
Newtonian structure eqn

Tolman-Oppenheimer-Volkoff (TOV) equation



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Three relativistic corrections:

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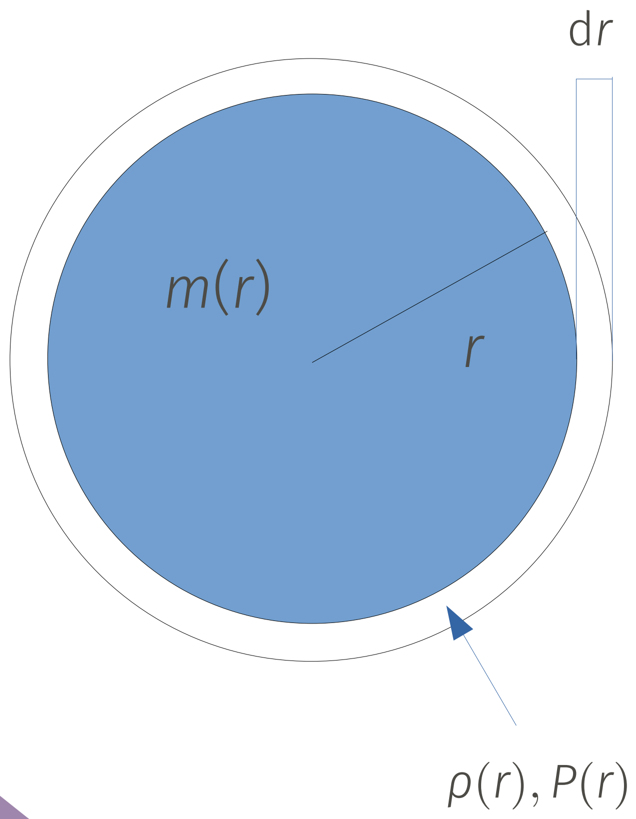

Three relativistic corrections:

1. Gravity is sourced by $m(r)$ and $P(r)$

$$m(r) \mapsto m(r) + 4\pi r^3 P(r)/c^2$$

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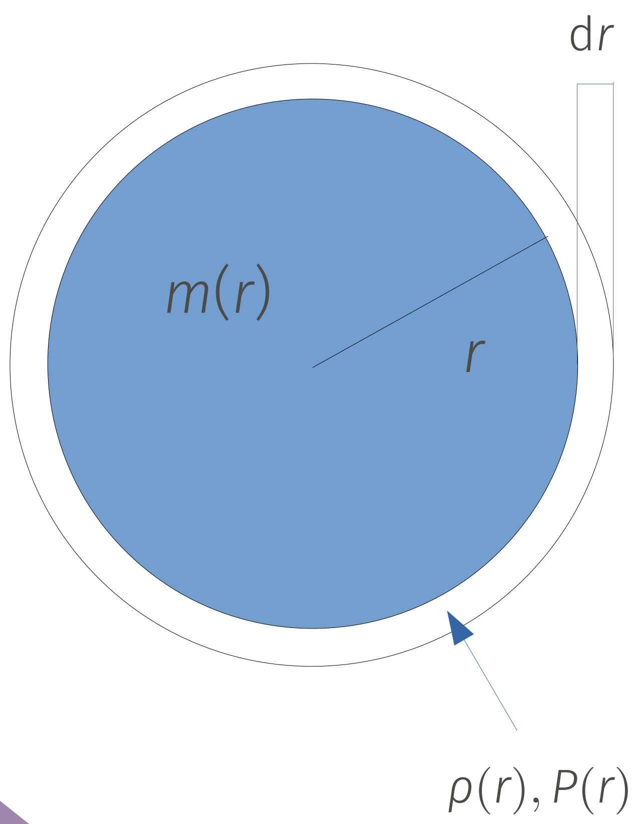

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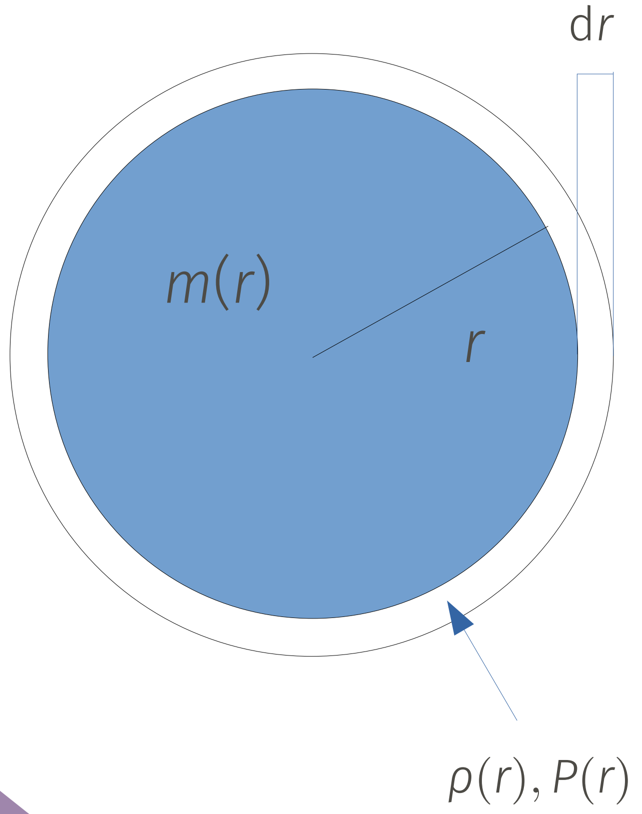

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$$r^2 \mapsto r^2\left[1 - 2Gm(r)/(rc^2)\right]$$

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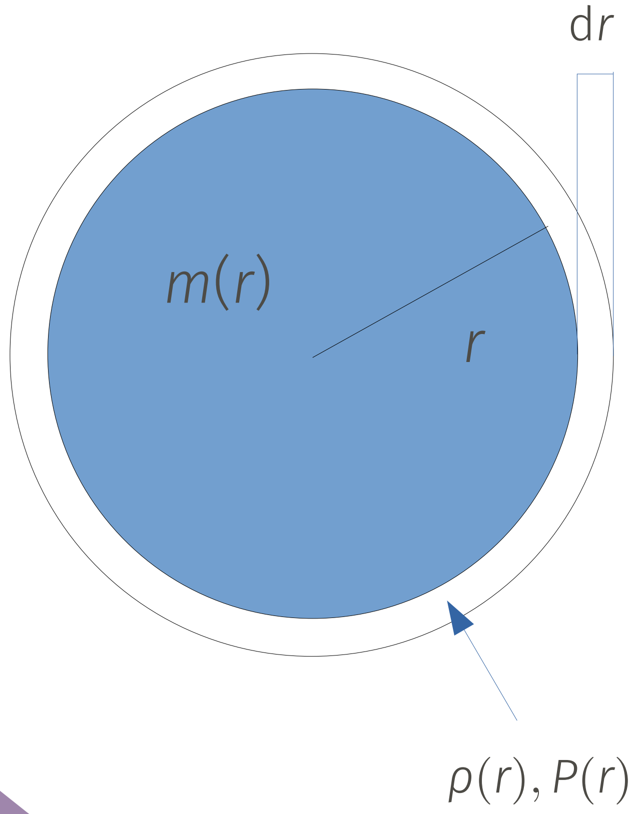

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TOV equation

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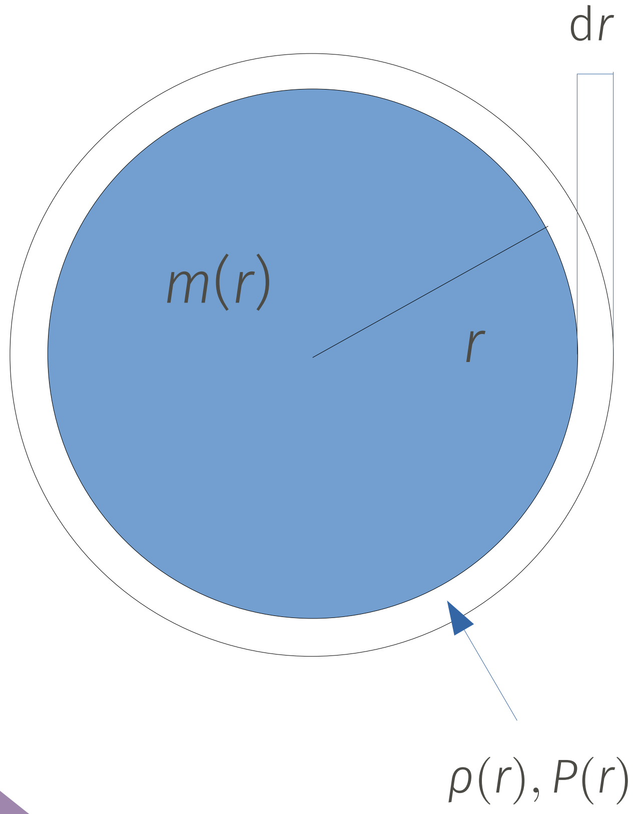

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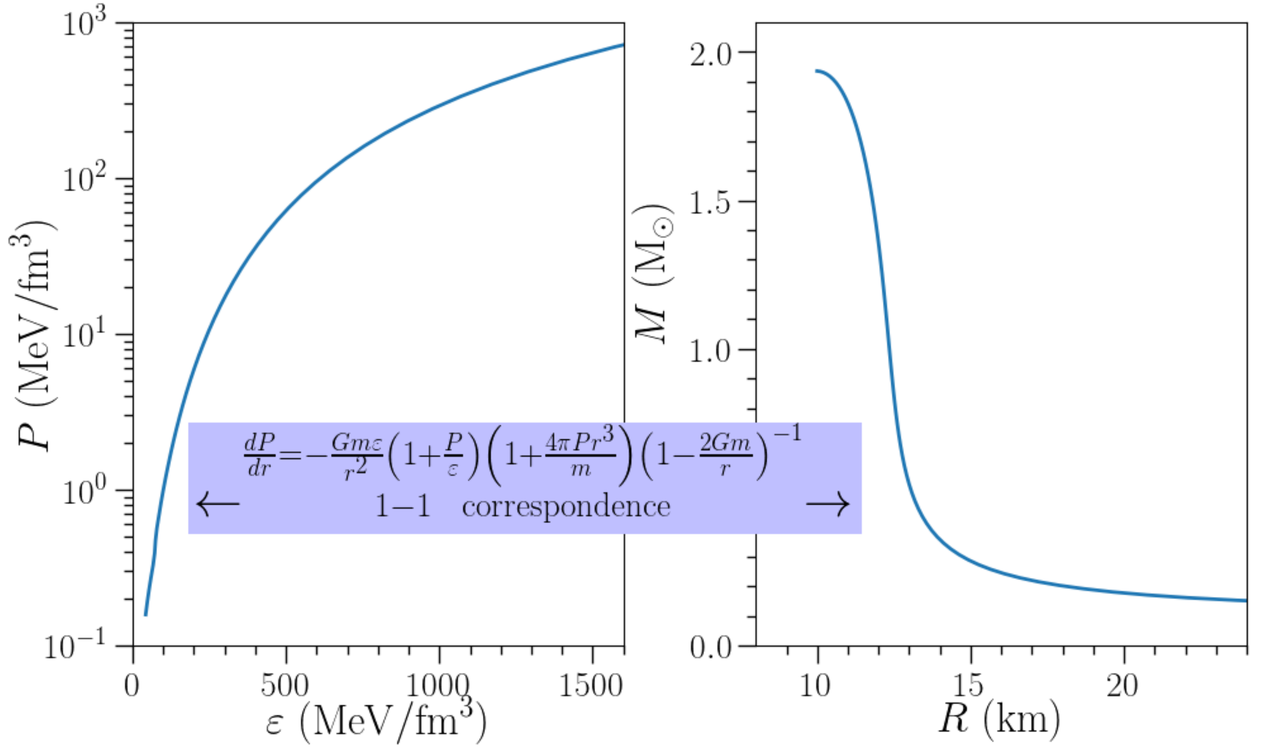
Supplement with equation of state (EOS) connecting p and ρ

TOV equation

$$\boxed{\frac{\mathrm{d}P}{\mathrm{d}r} = -\left[\rho(r) + \frac{P(r)}{c^2}\right]\frac{G\left[m(r) + 4\pi r^3 \frac{P(r)}{c^2}\right]}{r^2\left[1 - \frac{2Gm(r)}{rc^2}\right]}}$$

Tolman-Oppenheimer-Volkoff (TOV) equation

Microscopic physics can be constrained from *macroscopic* properties



Andrew Steiner

Neutron stars and the equation of state of dense matter

Tyler Gorda

TU Darmstadt

PhD Retreat, Graz (13-15.05.2022)

TECHNISCHE UNIVERSITÄT DARMSTADT

Lecture 2:
The EOS of Dense matter

Tyler Gorda

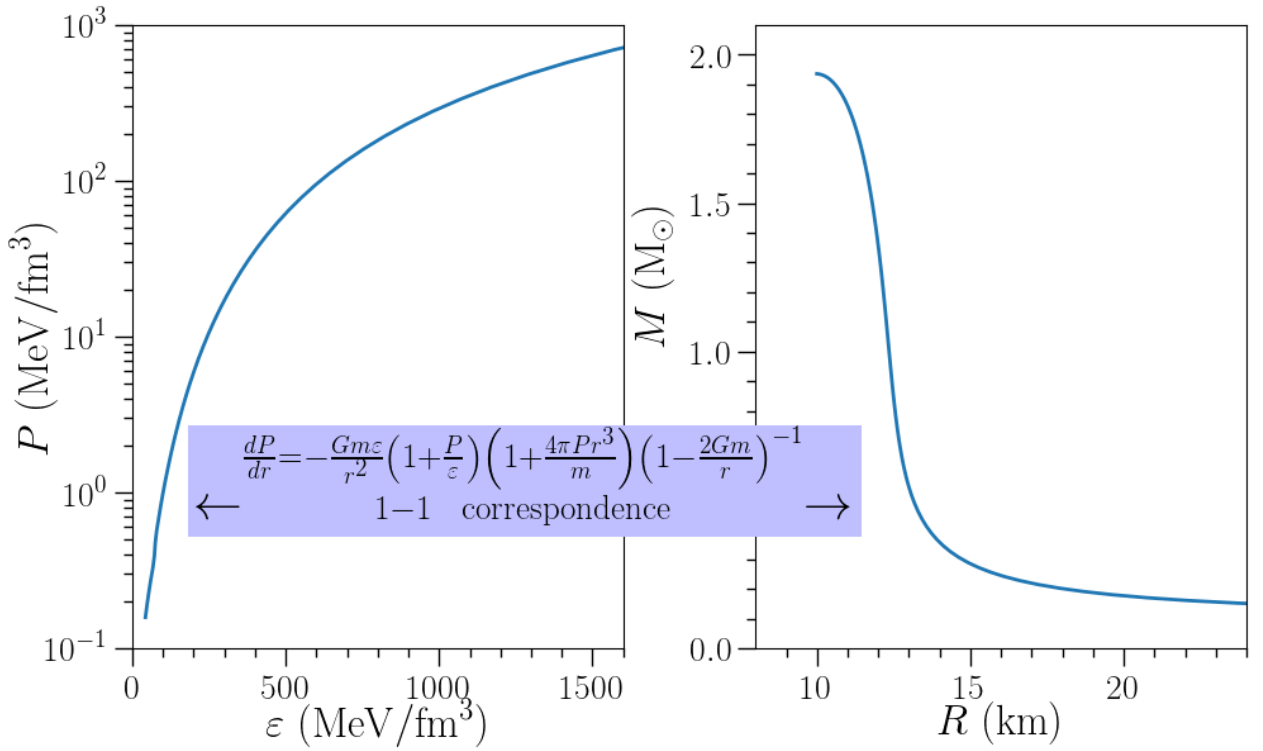
TU Darmstadt

PhD Retreat, Graz (13-15.05.2022)

TECHNISCHE
UNIVERSITÄT
DARMSTADT

Recap: TOV equations

Microscopic physics can be constrained from *macroscopic* properties



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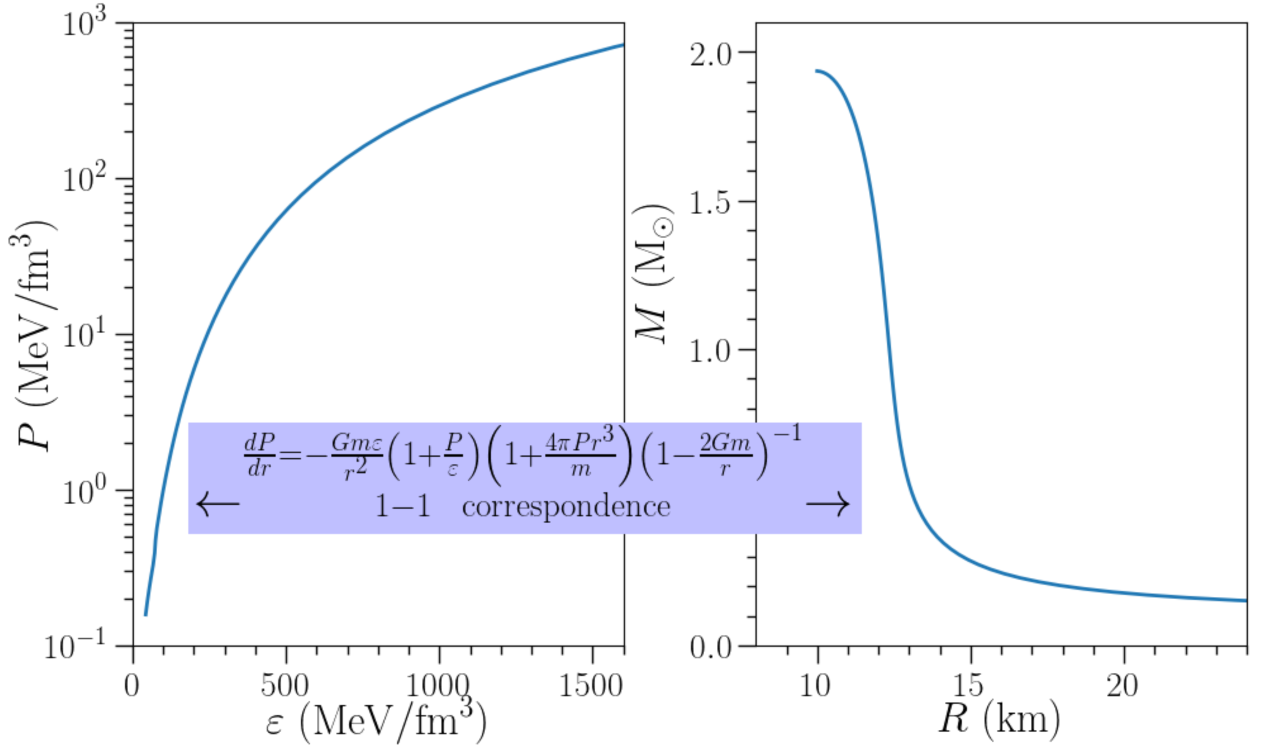
Need EOS of dense matter to predict...



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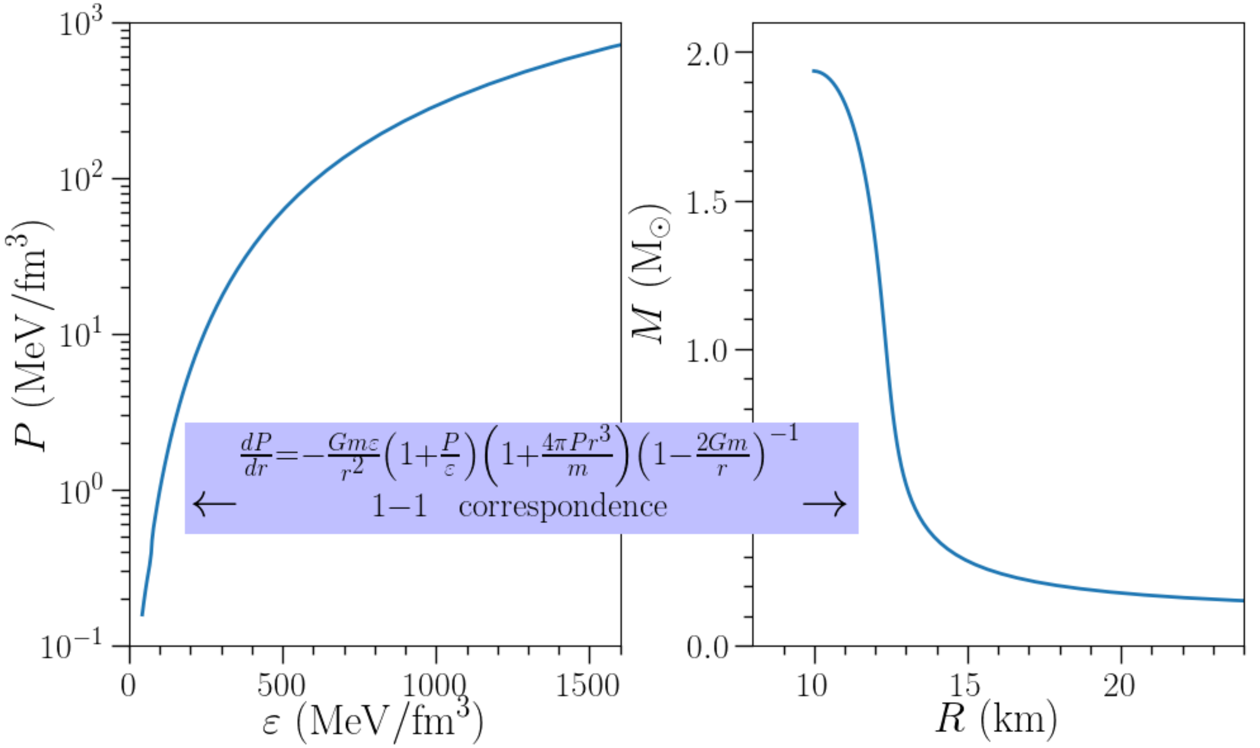


$$\frac{dP}{dr}=-\frac{Gm\varepsilon}{r^2}\left(1+\frac{P}{\varepsilon}\right)\left(1+\frac{4\pi Pr^3}{m}\right)\left(1-\frac{2Gm}{r}\right)^{-1}$$

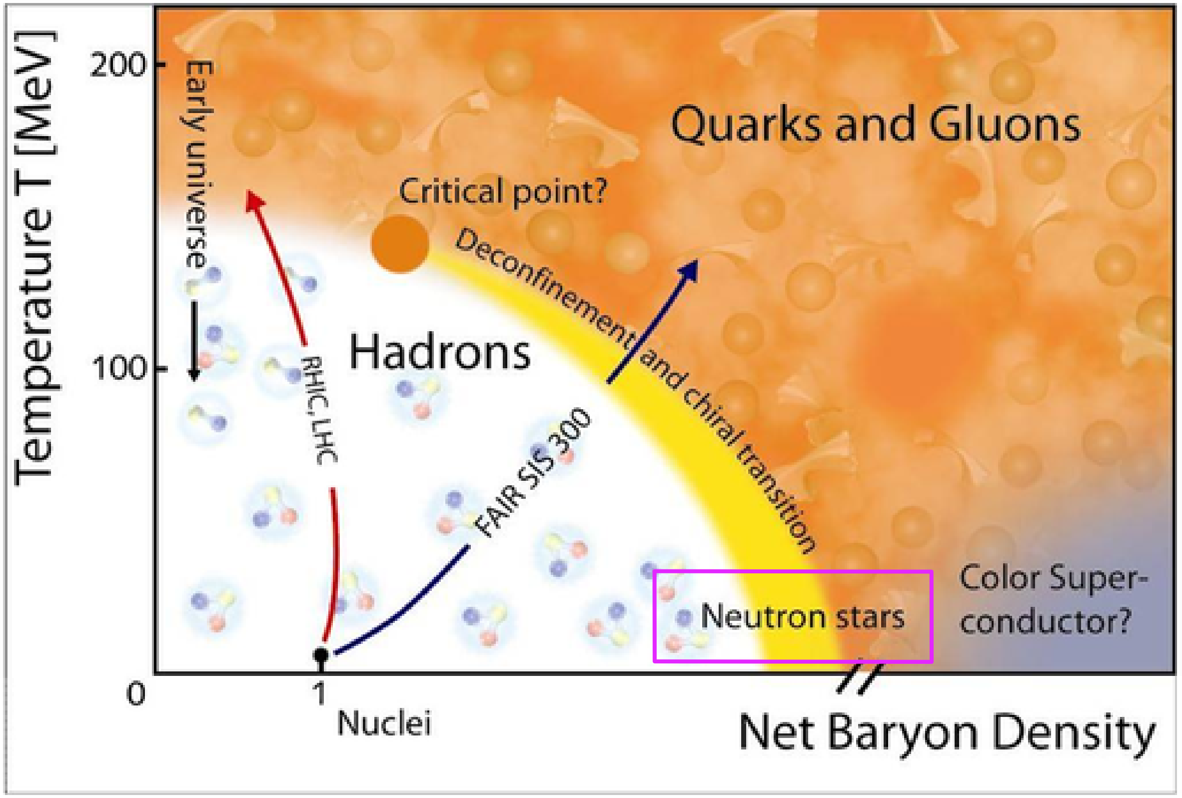
$\leftarrow$ 1–1 correspondence $\rightarrow$

Andrew Steiner

...or can use observations to constrain the EOS

Where does NS matter live in the phase diagram?

NSs probe densities beyond nuclear density, but below pQCD densities



Compressed Baryonic Matter (CBM) experiment

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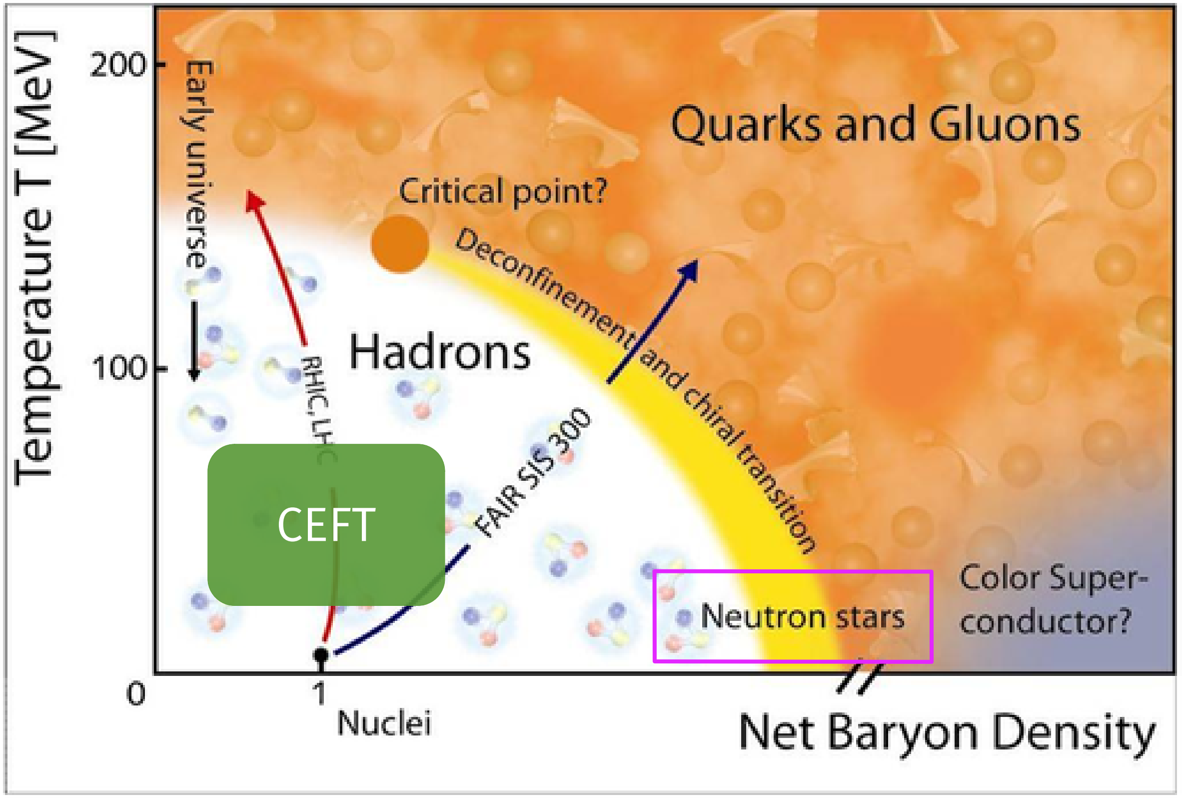

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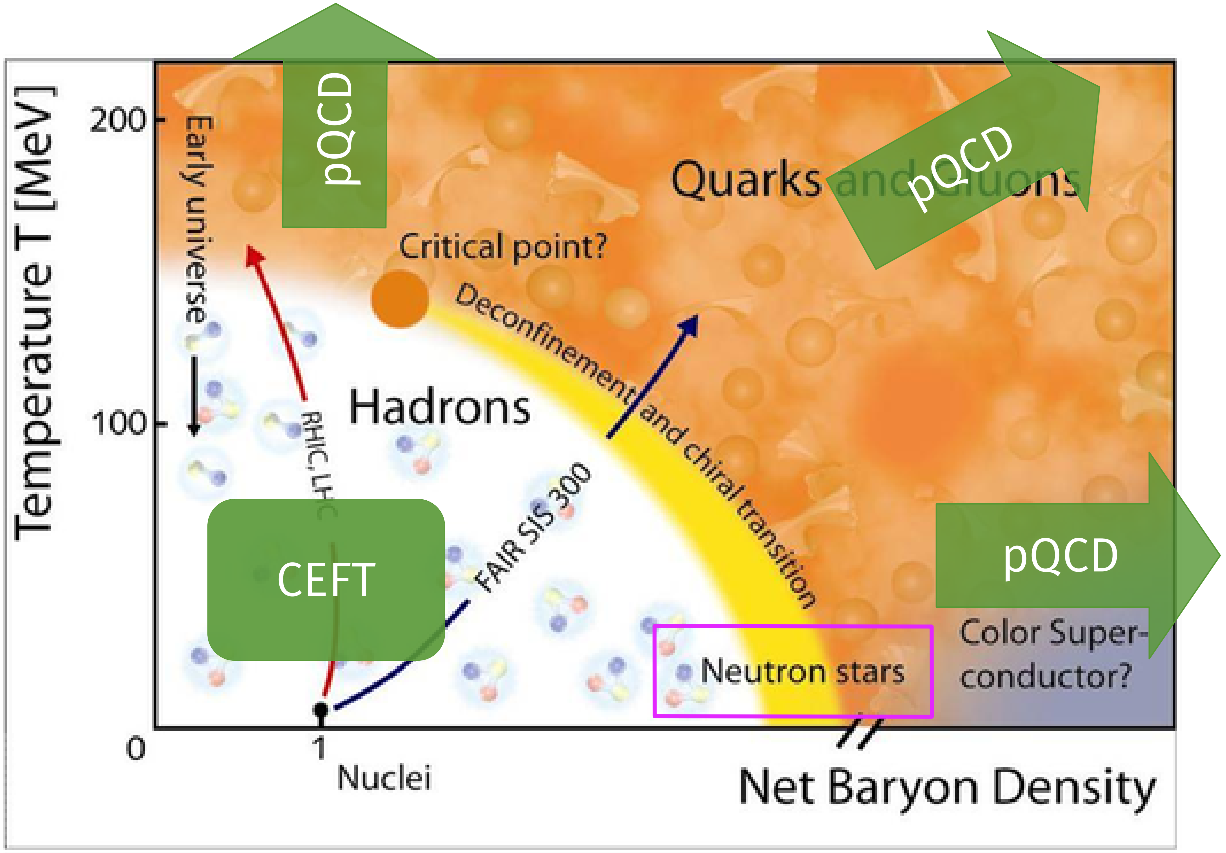

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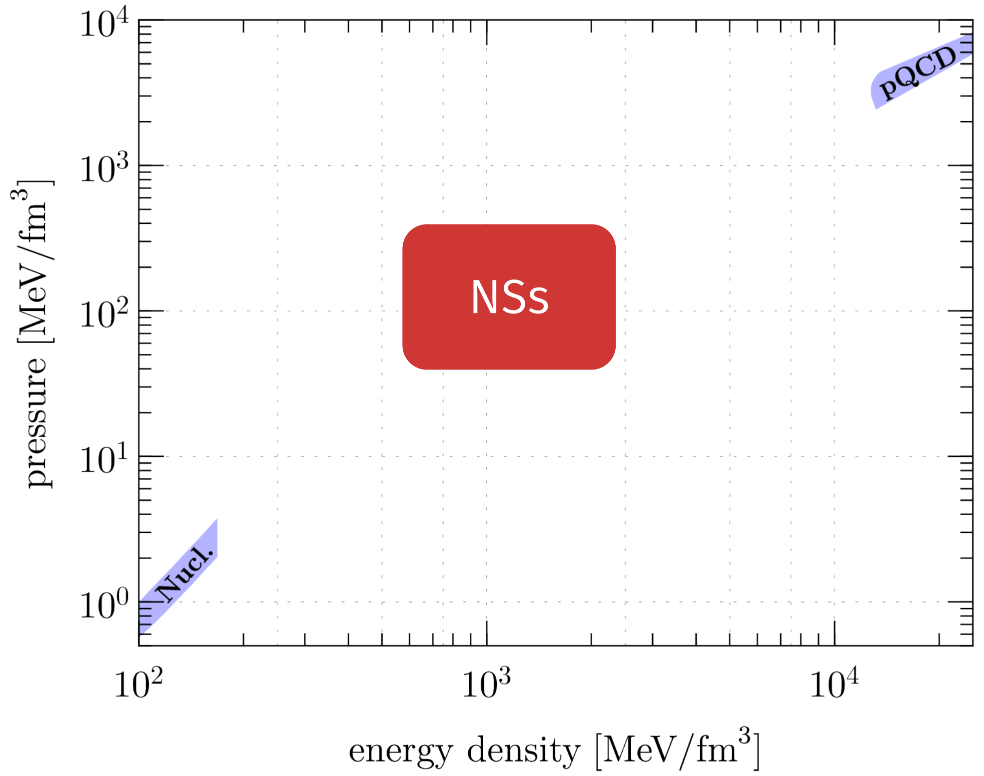

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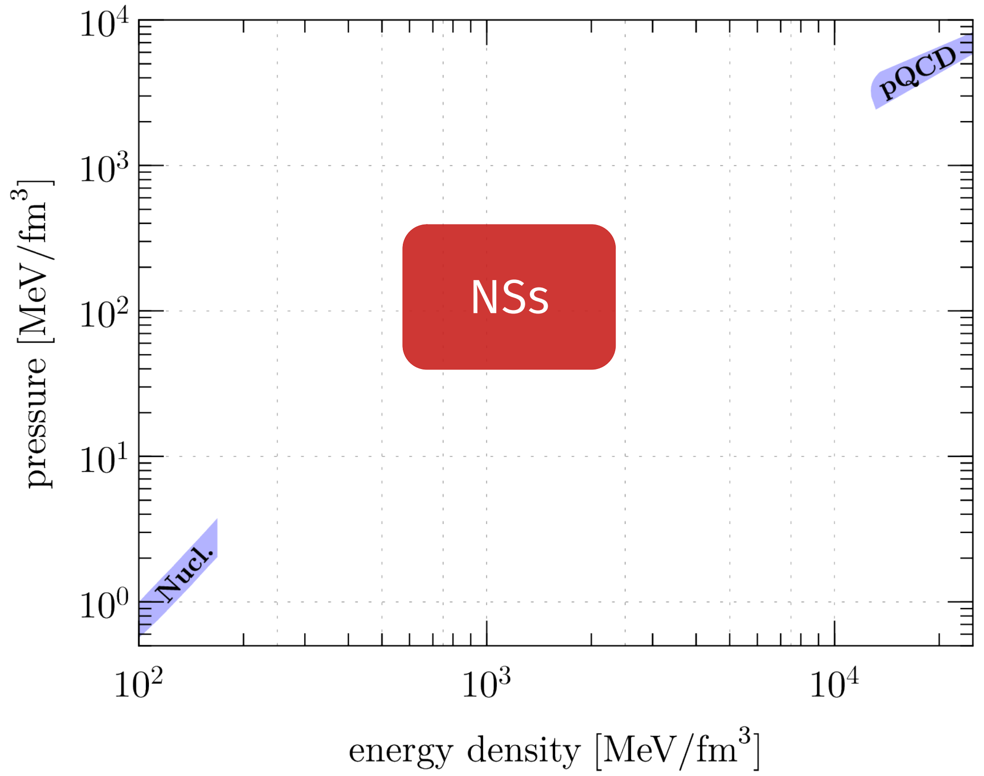

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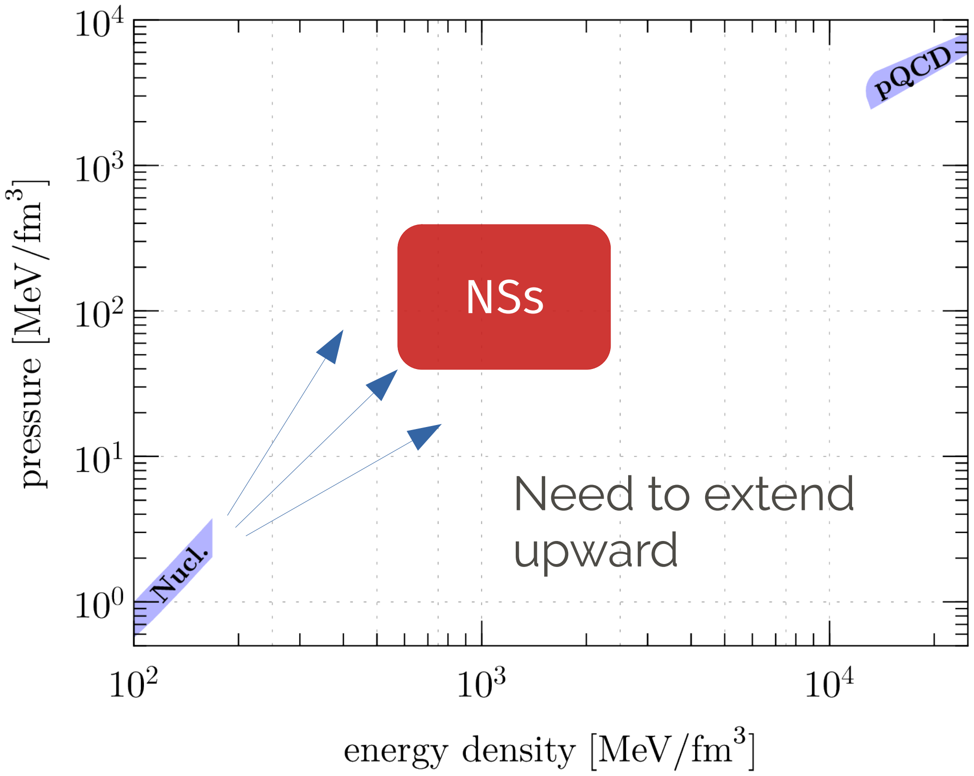

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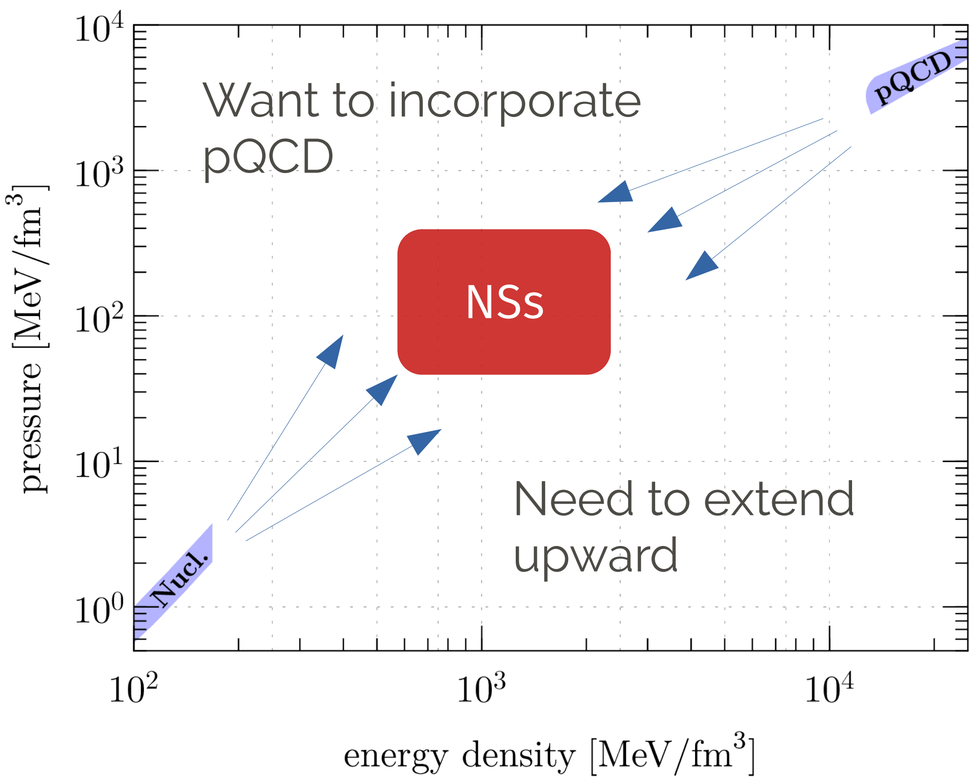

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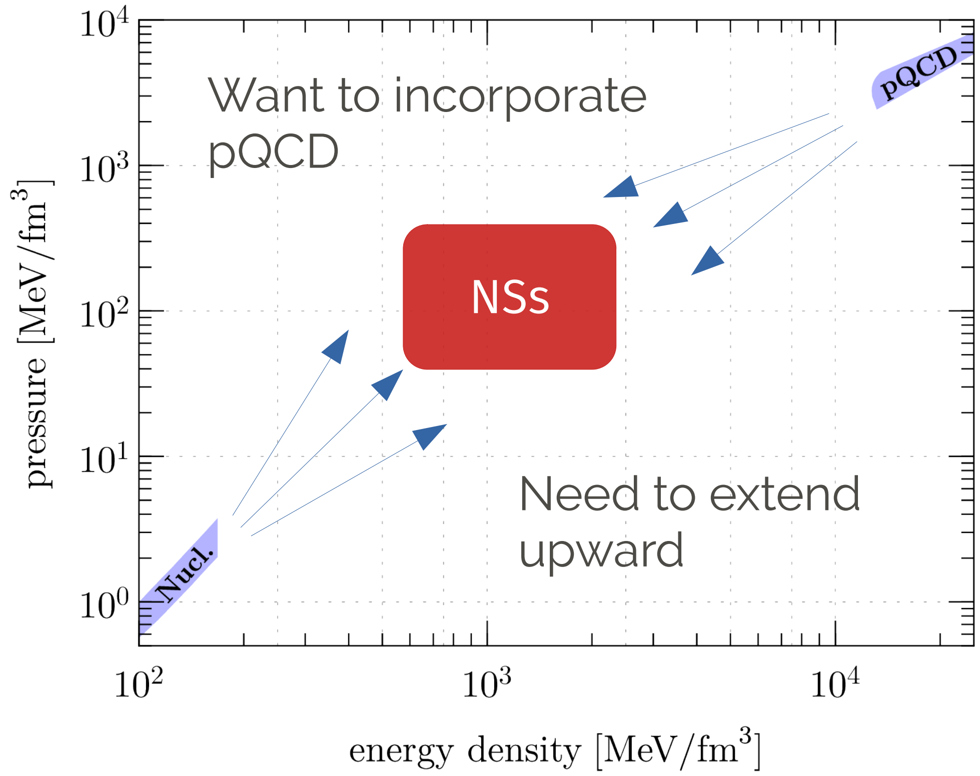

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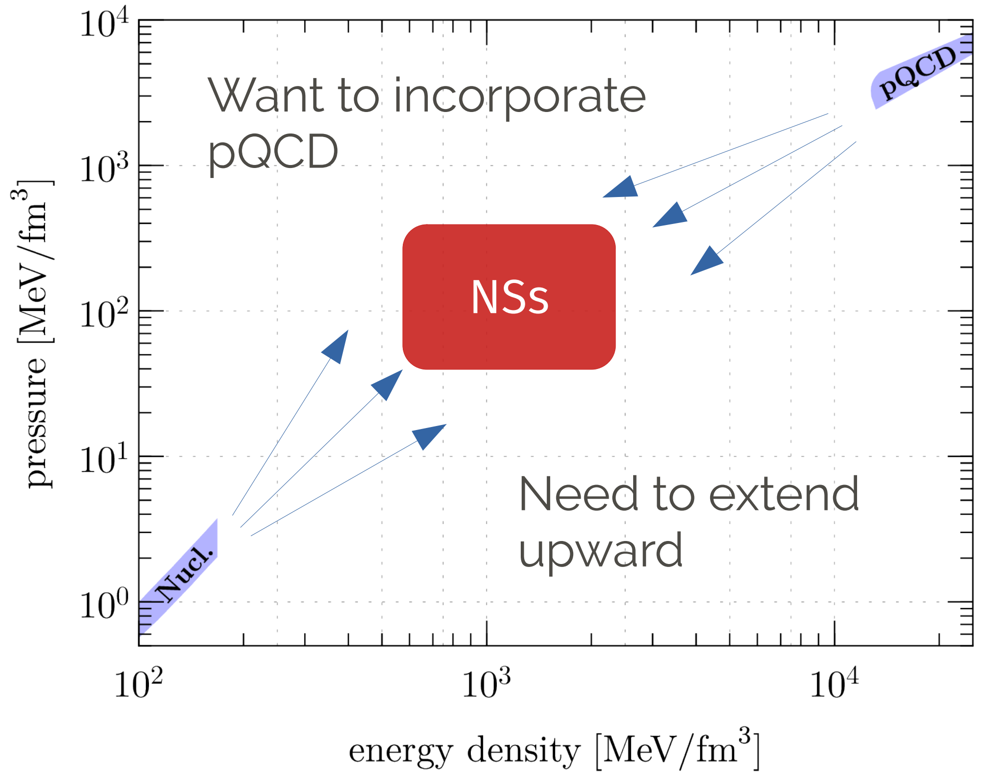

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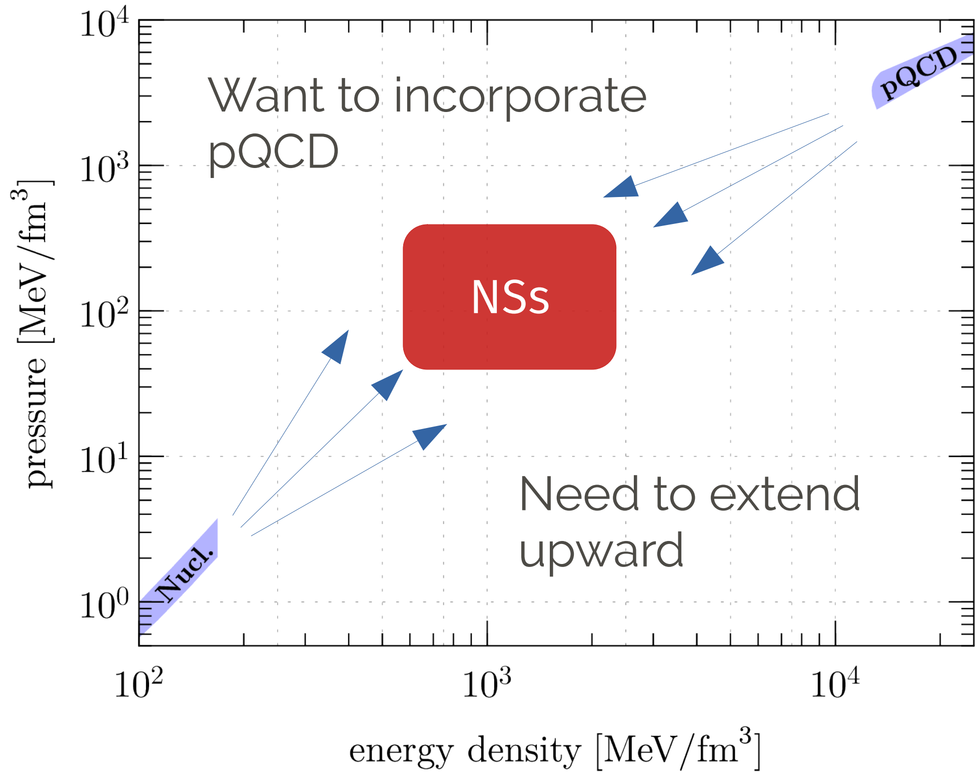

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* Topic of Lecture 3

Outline

1. General approach to EOS calculations
2. Overview of CET framework
3. Details pQCD and cold quark matter
    i. Overview
    ii. Infrared complications
    iii. State-of-the-art result

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Thermodynamics of QFTs: partition function (1/3)

Want to evaluate partition function:

conserved current

thermodynamic grand potential

$$Z = \underbrace{\mathrm{tr}\left[e^{-\beta\hat{H}}\right]}_{(T>0)} \rightarrow \underbrace{\mathrm{tr}\left[e^{-\beta(\hat{H}-\mu\hat{N})}\right]}_{(T>0,\mu>0)} = e^{-\Omega}$$

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Thermodynamics of QFTs: partition function (1/3)

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- Like in normal QFT, simplest to construct a ***path-integral*** representation of the partition function by dividing up the "time" interval into equal pieces:

$$e^{-\beta(\hat{H}-\mu\hat{N})} = \underbrace{e^{-\Delta\tau(\hat{H}-\mu\hat{N})}e^{-\Delta\tau(\hat{H}-\mu\hat{N})}\cdots e^{-\Delta\tau(\hat{H}-\mu\hat{N})}}_{N \text{ equal pieces}}, \quad \Delta\tau \equiv \frac{\beta}{N}$$

Thermodynamics of QFTs: partition function (2/3)

First we want to write the trace in the partition function in terms of an integral over states at the beginning and final "times":

$$Z = \mathrm{tr}\left[e^{-\beta(\hat{H}-\mu\hat{N})}\right] = \sum_{n>0} \langle n|e^{-\beta(\hat{H}-\mu\hat{N})}|n\rangle$$

$$= \int d(\varphi^\dagger, \varphi) e^{-\varphi^\dagger\varphi} \sum_{n>0} \underbrace{\langle n|\varphi\rangle}\, \langle\varphi|e^{-\beta(\hat{H}-\mu\hat{N})}|n\rangle$$

* these $|\varphi\rangle$ are "coherent states", but skipping details

$$|\varphi\rangle \equiv e^{\pm\varphi\hat{a}^\dagger}|0\rangle$$

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$$= \int d(\varphi^\dagger,\varphi)e^{-\varphi^\dagger\varphi}\langle\pm\varphi|e^{-\beta(\hat{H}-\mu\hat{N})}|\varphi\rangle$$

Bosons return to same field configuration; fermions to negative the field configuration!

Thermodynamics of QFTs: partition function (3/3)

Final result of the path-integral process is

$$
\begin{aligned}
Z &= \int d(\varphi^\dagger, \varphi) e^{-\varphi^\dagger \varphi} \langle \pm\varphi | e^{-\beta(\hat{H} - \mu \hat{N})} | \varphi \rangle \\
&= \int\limits_{\substack{\varphi^\dagger(\beta) = \pm\varphi^\dagger(0) \\ \varphi(\beta) = \pm\varphi(0)}} \mathcal{D}\varphi^\dagger(\tau) \mathcal{D}\varphi(\tau) \exp\left\{ -\int_0^\beta \mathrm{d}\tau \left[\varphi^\dagger(\tau) \frac{\mathrm{d}\varphi(\tau)}{\mathrm{d}\tau} + H[\varphi^\dagger(\tau), \varphi(\tau)] - \mu N[\varphi^\dagger(\tau), \varphi(\tau)] \right] \right\}
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For usual Hamiltonians, Legendre transformation gives a ***Lagrangian***:

$$\boxed{Z = \int_{\substack{\varphi^\dagger(\beta,\vec{x})=\pm\varphi^\dagger(0,\vec{x}) \\ \varphi(\beta,\vec{x})=\pm\varphi(0,\vec{x})}} \mathcal{D}\varphi^\dagger\mathcal{D}\varphi \exp\left\{-\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \left[\mathcal{L}_\mathrm{E} - \mu\mathcal{N}\right]\right\}}$$

Path integral: Main points (1/2)

$$\boxed{Z = \int_{\substack{\varphi^\dagger(\beta,\vec{x})=\pm\varphi^\dagger(0,\vec{x})\\ \varphi(\beta,\vec{x})=\pm\varphi(0,\vec{x})}} \mathcal{D}\varphi^\dagger \mathcal{D}\varphi \exp\left\{-\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x\, [\mathcal{L}_\mathrm{E} - \mu\mathcal{N}]\right\}}$$

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$$\text{e.g.} \quad \mathcal{L}^\mathrm{E}_\mathrm{QCD} = \sum_f \overline{\psi}^i_f \Big(\delta_{ij}(\gamma^\mathrm{E}_\mu \partial_\mu + m_f) - ig\gamma^\mathrm{E}_\mu A^a_\mu T^a_{ij}\Big)\psi^j_f + \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu},$$

Path integral: Main points (2/2) – High density

$$\boxed{Z = \int\limits_{\substack{\varphi^\dagger(\beta,\vec{x})=\pm\varphi^\dagger(0,\vec{x}) \\ \varphi(\beta,\vec{x})=\pm\varphi(0,\vec{x})}} \mathcal{D}\varphi^\dagger \mathcal{D}\varphi \exp\left\{-\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \left[\mathcal{L}_\mathrm{E} - \mu\mathcal{N}\right]\right\}}$$

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- Chemical potential is like imaginary A^0 field in Euclidean Lagrangian

$$\mathcal{L}_E \ni (\overline{\psi}\partial\!\!\!/_\mu\psi) - (\overline{\psi}\gamma^0_\mathrm{E}\psi)\mathrm{i}gA^0 \quad \longleftrightarrow \quad (\overline{\psi}\partial\!\!\!/_\mu\psi) - (\overline{\psi}\gamma^0_\mathrm{E}\psi)\mu$$

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$$\boxed{\mathrm{i}\omega_n \mapsto \mathrm{i}\omega_n - \mu = \mathrm{i}(\omega_n + \mathrm{i}\mu) \text{ imaginary shift to the frequency!}}$$

Outline

1. General approach to EOS calculations
2. **Overview of CET framework**
3. Details pQCD and cold quark matter
   1. Overview
   2. Infrared complications
   3. State-of-the-art result

Chiral Perturbation theory

Perturbative EFT of low-energy QCD that respects the chiral symmetry of the fundamental theory.

(Chiral symmetry of QCD holds with massless (u, d) quarks – it is the invariance of the theory under isospin transformations between (u, d).)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \cdots$$

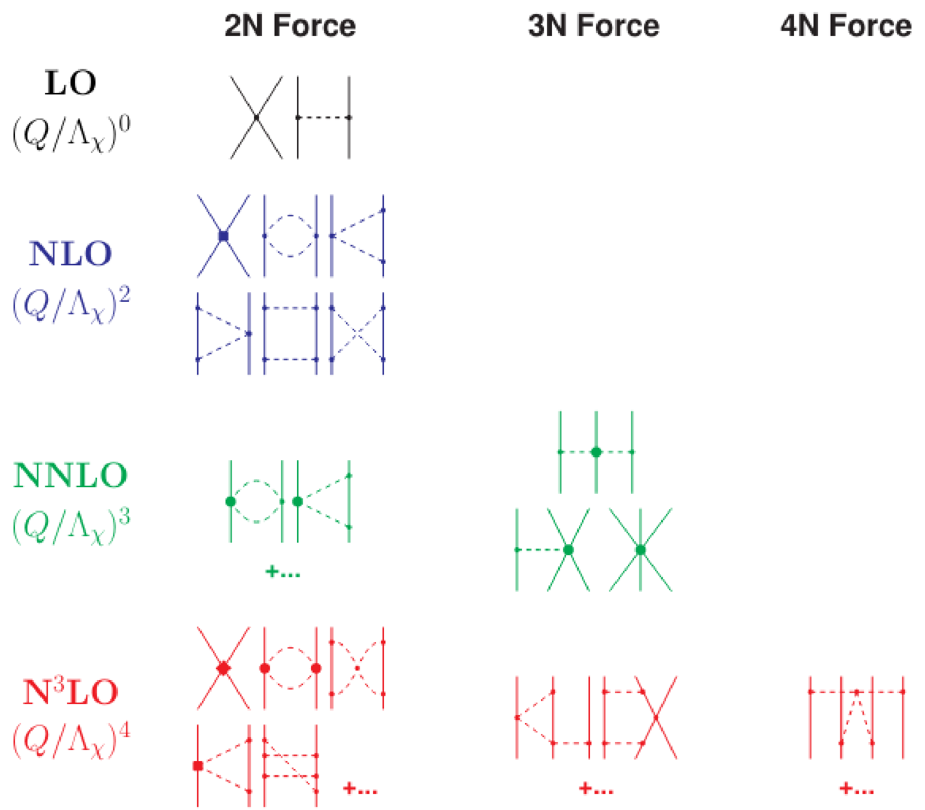
Pion-pion interactions

Pion-nucleon interactions

nucleon-nucleon interactions

Chiral Perturbation theory

EFT in low-momentum expansion; terms grouped by powers of $(Q/\Lambda)^k$, with Λ the breakdown scale. E.g.:



Machleidt & Entem Phys.Rept. 503 (2011)

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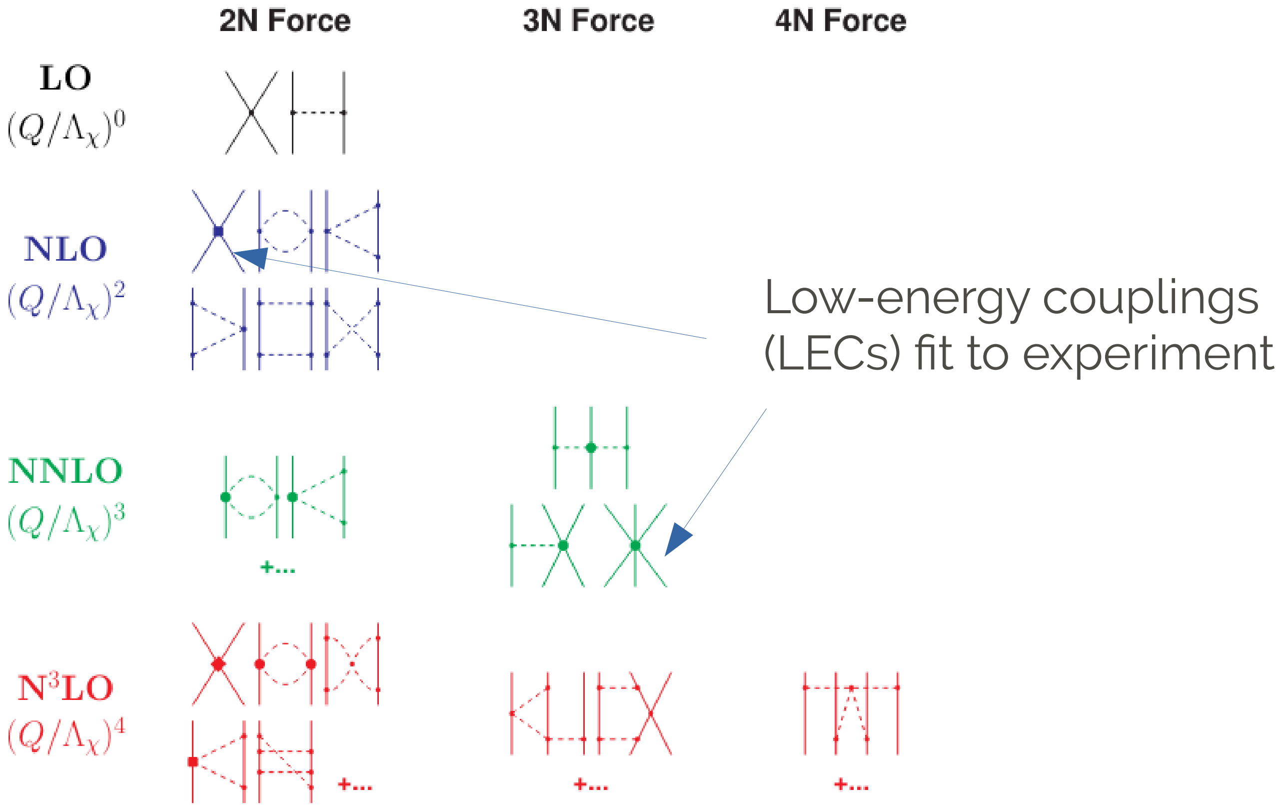
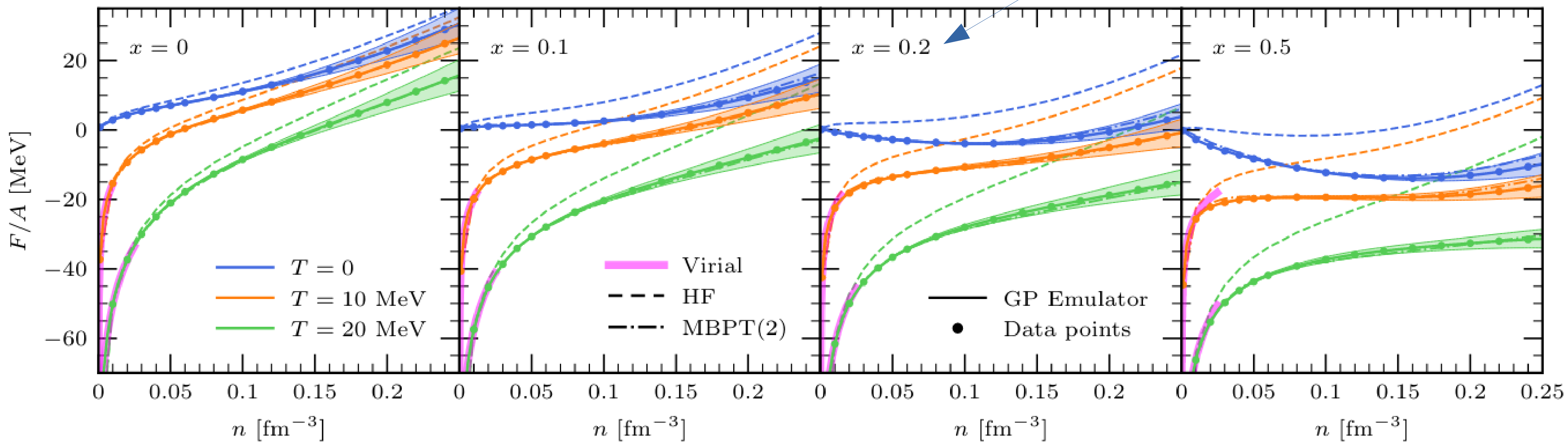


Machleidt & Entem Phys.Rept. 503 (2011)

Chiral Perturbation theory

Can calculate the EOS for low density and temperature

Proton fraction $x = n_p / n$



Keller, Hebeler, Schwenk arXiv:2204.14016

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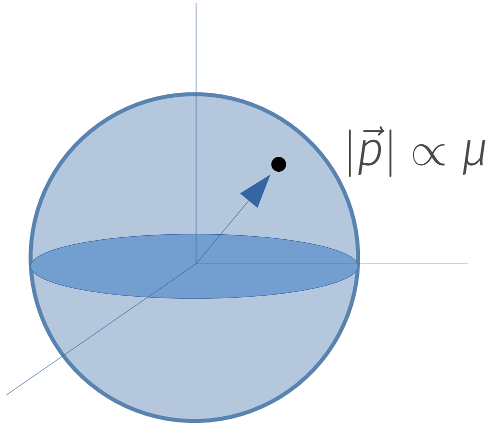
Cold QM and pQCD overview 1/2

Basic property of cold QM EoS is that it's approximately described by a free quark gas



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- *QM has colored quarks/gluons as DOF*
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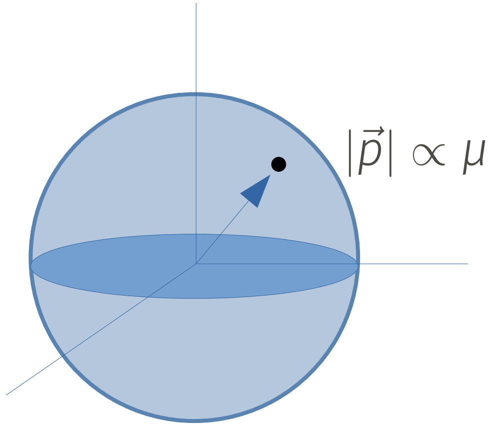

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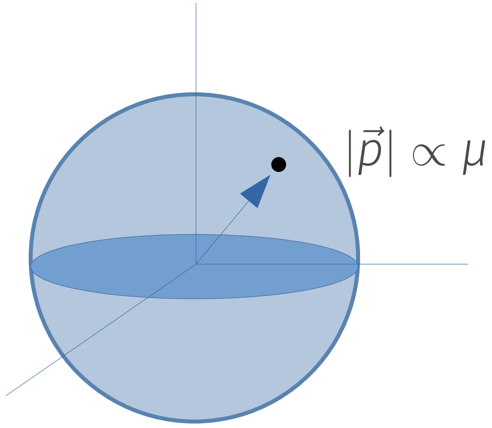
  $^{*}(p_{\mathrm{FD}} \propto \mu^4, \quad p_{\mathrm{pairing}} \propto \mu^2\Delta^2)$

  Alford+, Rev. Mod. Phys. 80, 1455 (2008)



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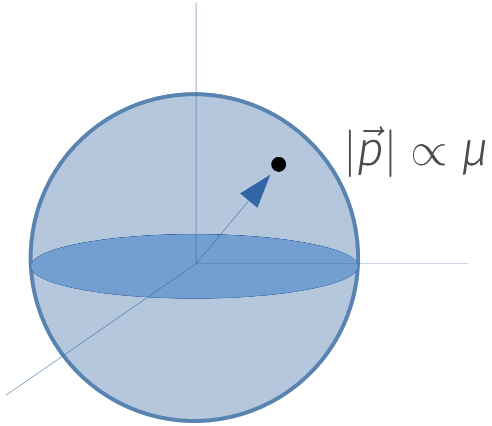
$\implies \varepsilon = 3p$



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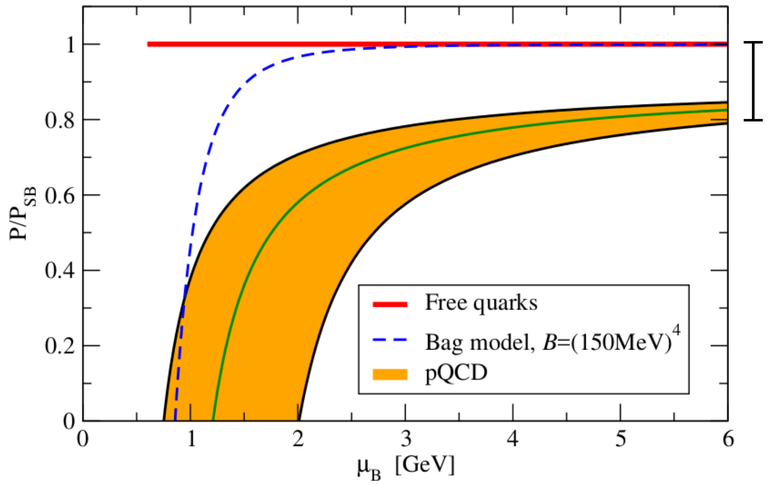

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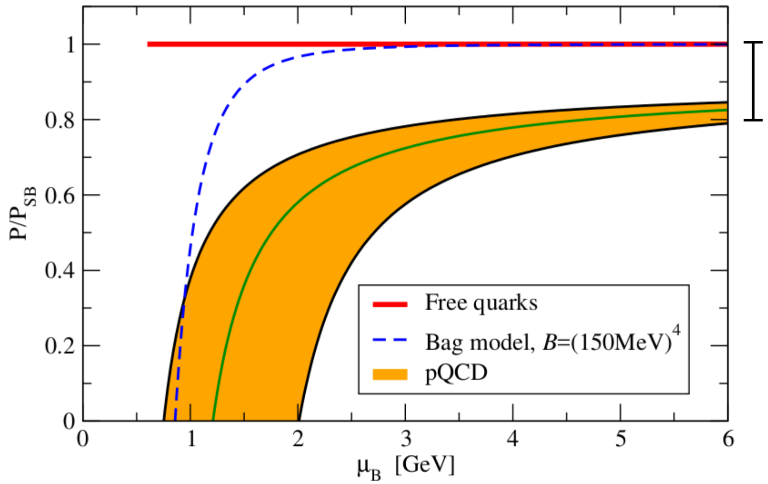

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So we want to calculate these corrections accurately!

Cold QM and pQCD overview 2/2

Framework for cold QM computations is relativistic thermal QFT.

- Systemmatic framework for calculating corrections in a series expansion in α_s*
  (*important caveats to come!*)

$$p = \underbrace{p_0}_{\textit{free quark gas}} + p_1\alpha_s + p_2\alpha_s^2 + \cdots$$

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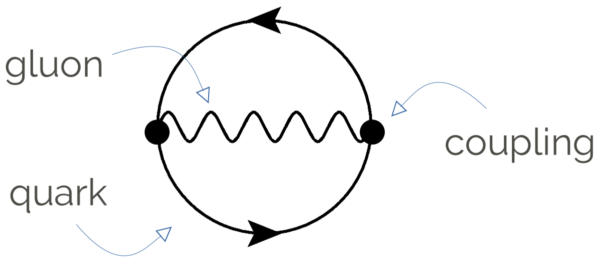

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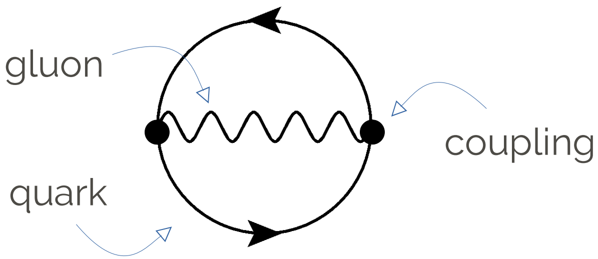
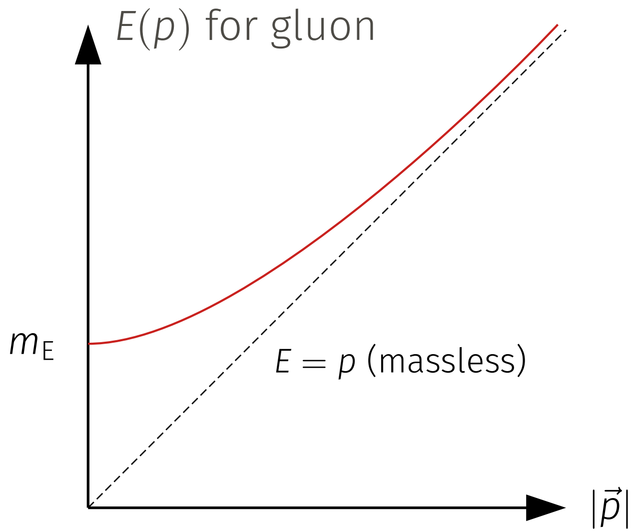


*(no external lines because this is the vacuum with μ>0)

IR complications within calculations: 1/4

Important caveat is that TQFT has IR (long-wavelength) differences from what you would expect

e.g.:



$$-E(\vec{p})^2 + \vec{p}^2 + \overbrace{\Pi\big(E(\vec{p}),\vec{p}\big)}^{\text{“self-energy”}} = 0$$

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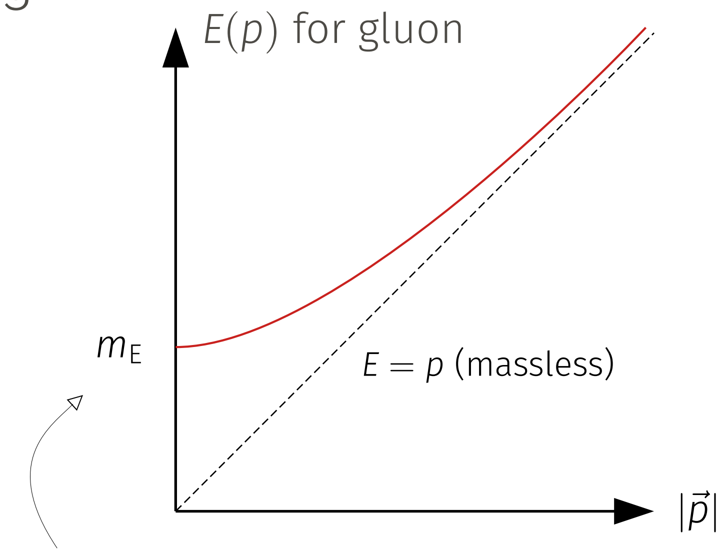

gluon has a ***thermal mass***!

$$m_E = O(\alpha_s^{1/2}\mu, \alpha_s^{1/2}T)$$

→ leads to screening of gluon modes

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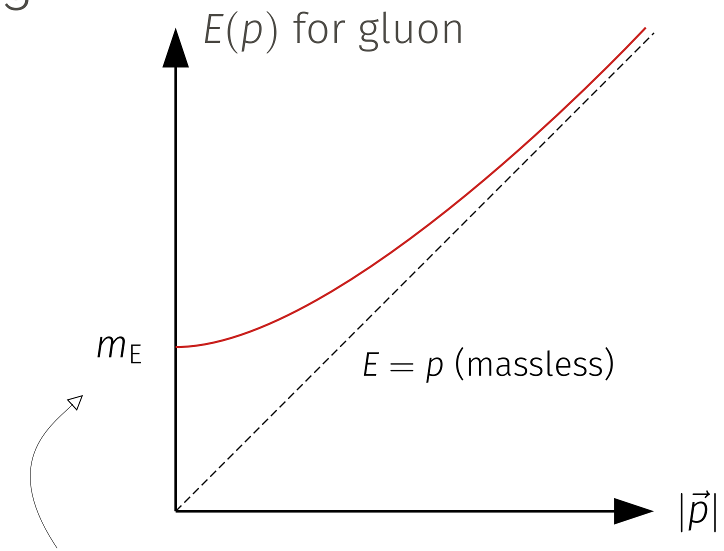

gluon has a ***thermal mass***!

$$m_{\mathrm{E}} = O(\alpha_s^{1/2}\mu, \alpha_s^{1/2}T)$$

→ leads to screening of gluon modes

$$-E(\vec{p})^2 + \vec{p}^2 + \overbrace{\Pi\big(E(\vec{p}), \vec{p}\big)}^{\text{“self-energy”}} = 0$$

*describes quantum + statistical corrections to particle propagation

- Mass screws up naive Feynman-diagram expansions

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IR complications within calculations: 1/4

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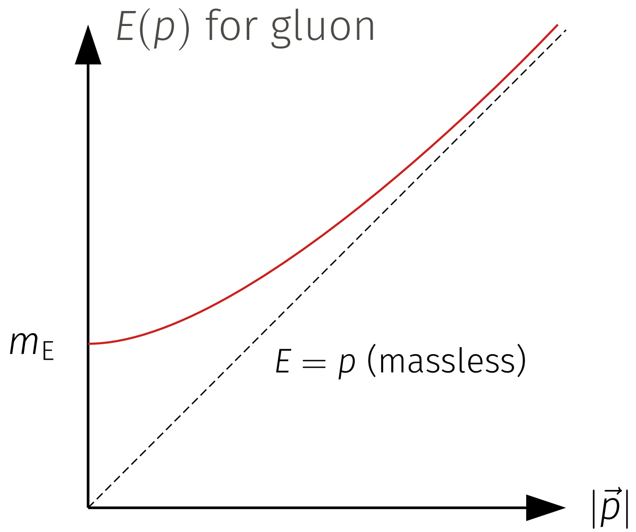

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“Hard thermal/dense loops”

Braaten & Pisarski, Phys. Rev. D 42 (1990), 46 (1992);
in cold QM context: Manuel, Phys. Rev. D 53 (1996)

IR complications within calculations: 2/4

Gluon dispersion relation:

$$\underbrace{-\omega^2 + \vec{k}^2} + \overbrace{\Pi(\omega, \vec{k})}^{O(g^2\mu^2)} = 0$$

Expression for self-energy is dominated by large-momentum quantum + statistical fluctuations

when $\omega, \vec{k} \sim g\mu$, can't ignore self energy

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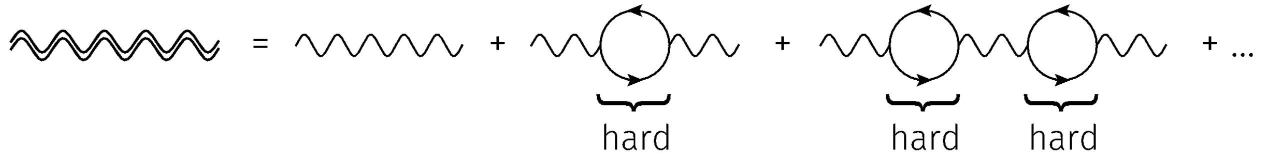

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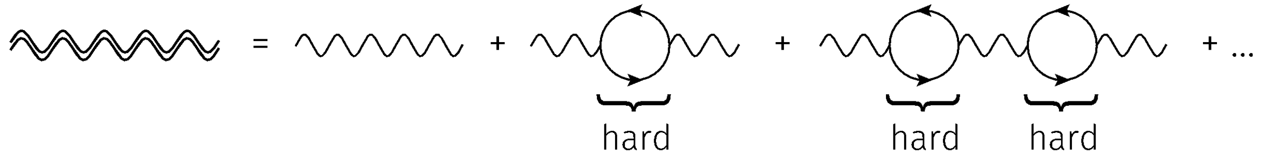

* also have corrected vertices

Self-energy evaluation; Hard Thermal/Dense Loop limit (1/3)

The self energy has a nontrivial IR limit; let's look a little at the calculation in QCD:

$$\Pi(P) = g^2 T_f \delta^{ab} \langle (\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma^\nu\psi)\rangle_{0,c} \quad = -g^2 T_f \delta^{ab} \mathrm{tr}[\langle \psi\bar{\psi}\rangle_0 \gamma^\mu \langle \psi\bar{\psi}\rangle_0 \gamma^\nu]$$

(only connected contraction; reordered the fermions)

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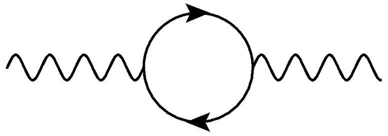

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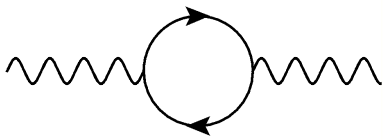

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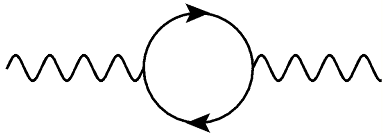
$=$ 

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Now look at low-momentum limit of this expression

Self-energy evaluation; Hard Thermal/Dense Loop limit (2/3)

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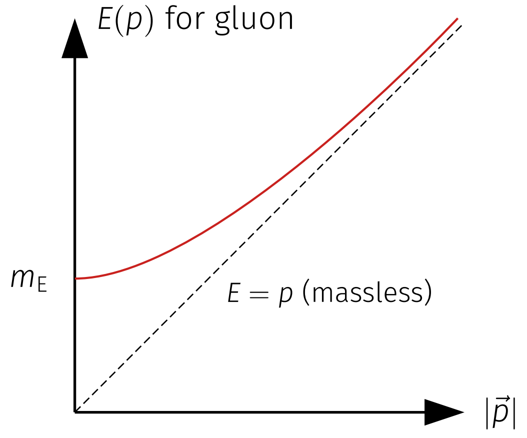

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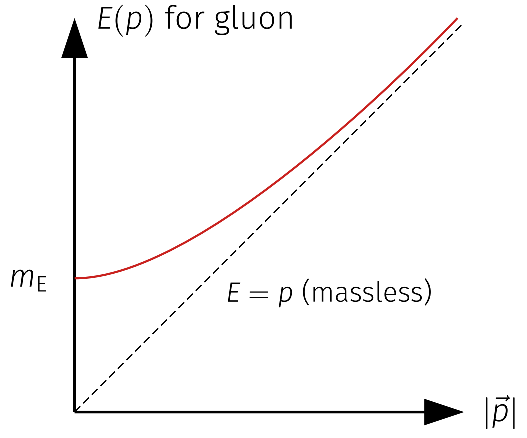

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Self-energy evaluation; Hard Thermal/Dense Loop limit (3/3)

Nontrivial dependence on $P^0/|\vec{p}|$ in the HTL result (so more than just a thermal mass):

$$\Pi^{\mu\nu}_{ab}(P) = m_{\mathrm{E}}^2 \int_{\hat{v}} \left(\delta^{\mu 0}\delta^{\nu 0} - \frac{\mathrm{i}P^0}{P \cdot V} V^\mu V^\nu \right)$$

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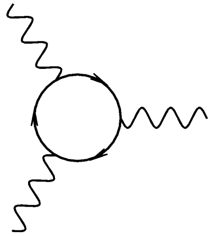
Similar HTL contributions for N-point gluon functions:



IR complications within calculations: 3/4

Hot QGP

Three scales:

1) $P \sim T$: Naive (hard) diagrams
2) $P \sim \alpha_s^{1/2} T$: EFT for (massive) chromo-electric fields
3) $P \sim \alpha_s T$: Lattice EFT for (massless) chromo-magnetic fields

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Cold QM

Pressure sum of two pieces:

1) $P \sim \mu$: Naive (hard) diagrams
2) $P \sim \alpha_s^{1/2} \mu$: massive gluonic fields, but **no simple EFT**

No softer scale b/c gluons not thermally occupied at T = 0: Great!

IR complications within calculations: 3/4

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Not great

IR complications within calculations: 4/4

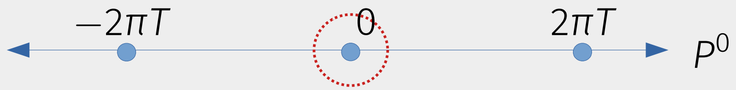
Effective in-medium mass scale: $m_{\mathrm{E}} \text{“} = \text{”} \lim_{|P|\to 0} \Pi^{\mu\nu}(P)$

$m_{\mathrm{E}} \sim \alpha_s^{1/2}\mu$

(coeff in front; also angular function)

Hot QGP

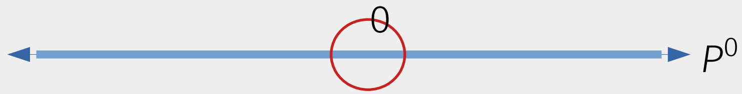
sum-integrals: $$T\sum_{n=-\infty}^{\infty}\int \frac{\mathrm{d}^3P}{(2\pi)^3}$$



- Only zero-mode requires special treatment for $m_{\mathrm{E}} << T$
- 3d EFT of massive zero mode “dimensional reduction”

Cold QM

4d-Euclid. Integrals: $$\int \frac{\mathrm{d}^4P}{(2\pi)^4}$$



- No simple separation
- No simple EFT to deal with IR problems

Current state-of-the-art pQCD EOS: 1/3

All of this modifies naive expectations. Current state-of-the-art: contributions from different kinematic regions

$$p = \boxed{\underbrace{p_0}_{\textit{free quark gas}} + p_1^h \alpha_s + p_2^h \alpha_s^2 + p_3^h \alpha_s^3} \quad \leftarrow \text{ scale } |P| \gtrsim \mu$$

$$\boxed{+ \underbrace{p_2^s \alpha_s^2}_{\substack{\textit{free soft} \\ \textit{pressure} \\ \textit{(screened)}}} + p_3^s \alpha_s^3} \quad \leftarrow \text{ scale } |P| \lesssim m_E$$

$$\boxed{+ p_3^m \alpha_s^3} \quad \leftarrow \textit{mixed}\text{; both scales}$$

TG+ Phys. Rev. D 104 (2021), Phys. Rev. Lett. 127 (2021); TG+ 2204.11893, 2204.11279;
see also TG+ Phys. Rev. Lett. 121 (2018); $O(\alpha_s^2)$: Freedman & McLerran Phys. Rev. D 16 (1977)

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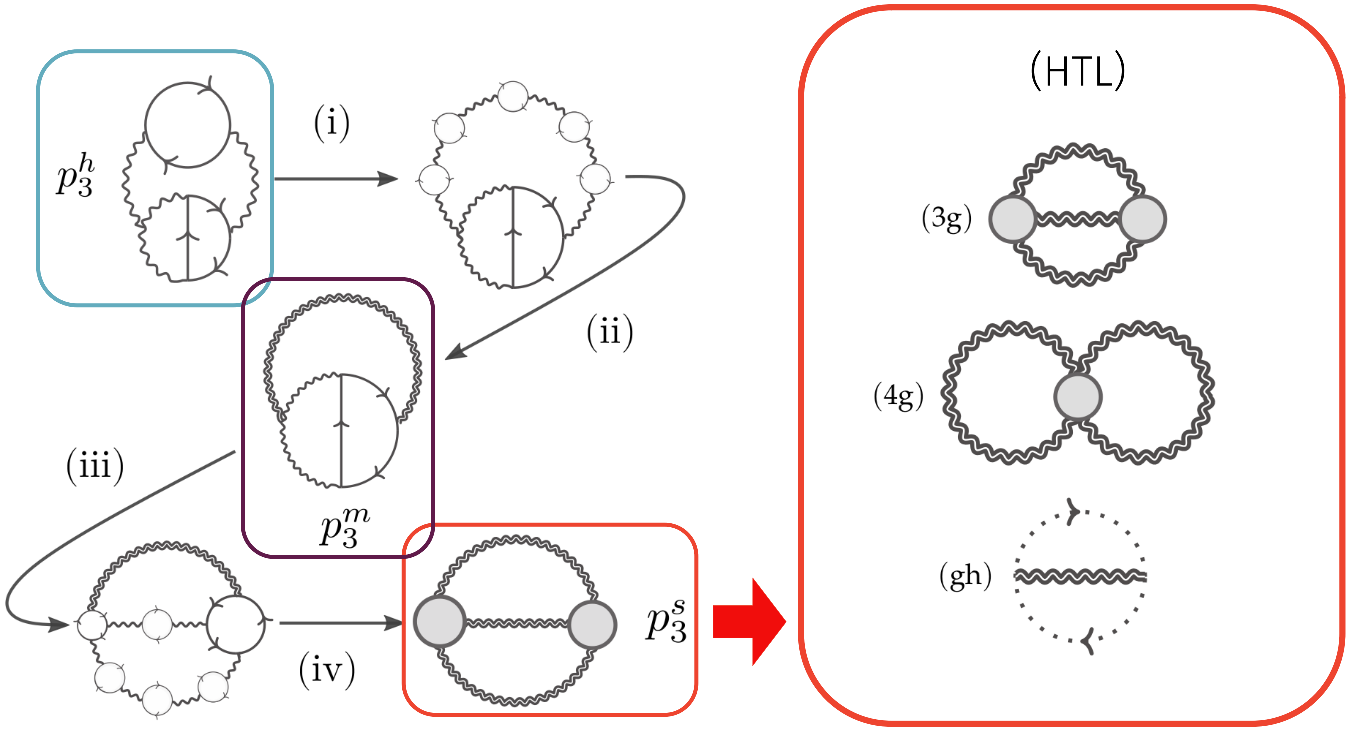
**Ambiguity in soft/hard split ($m_E \ll K \ll \mu$) gives logarithmic sensitivity to a factorization mass scale Λ_h, which cancels out of sum over all kinematic regions (columns!)

Current state-of-the-art pQCD EOS: 2/3

Current state-of-the-art: have now computed N³LO contributions from *HTL effective theory*

TG, Kurkela, Paatelainen, Säppi, Vuorinen, Phys. Rev. Lett. 127 (2021), Phys. Rev. D 104 (2021)



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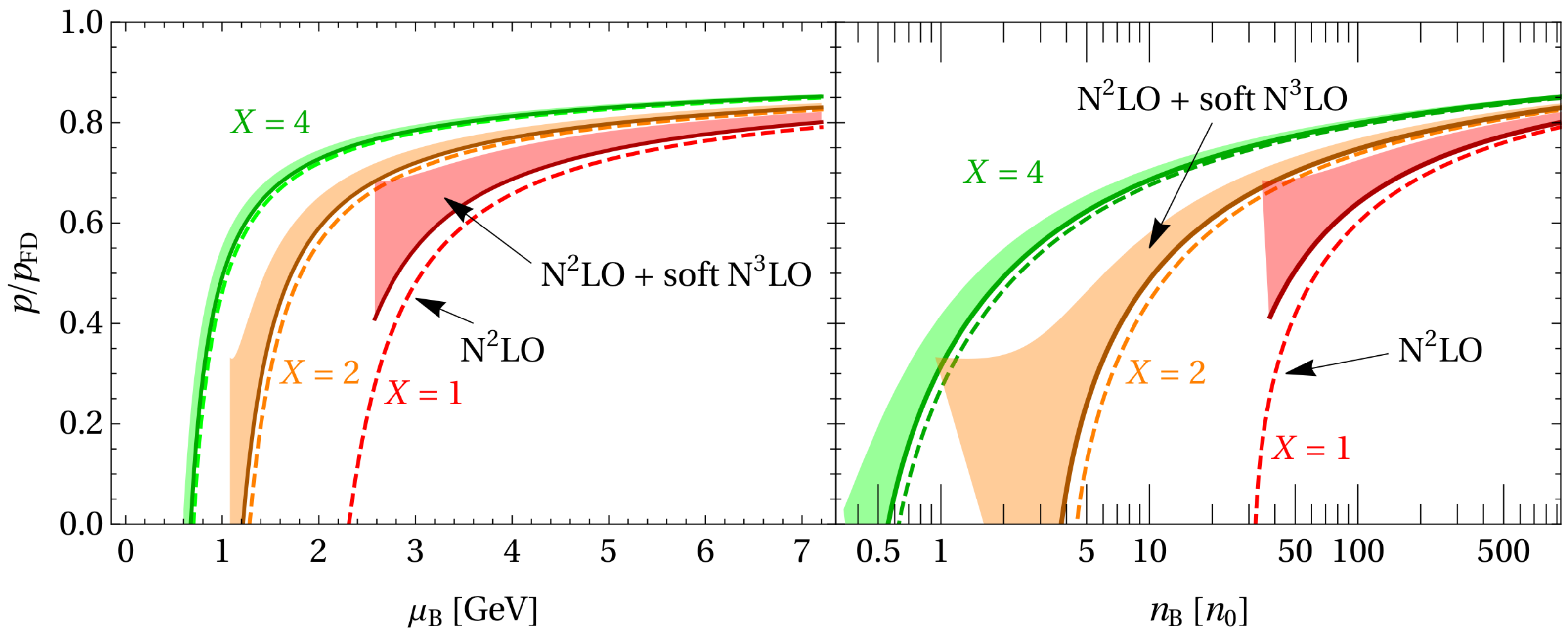

Decreases renormalization-scale sensitivity

Neutron stars and the equation of state of dense matter

Tyler Gorda

TU Darmstadt

PhD Retreat, Graz (13-15.05.2022)

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Lecture 3:
Constraining the NS-matter EOS

Tyler Gorda

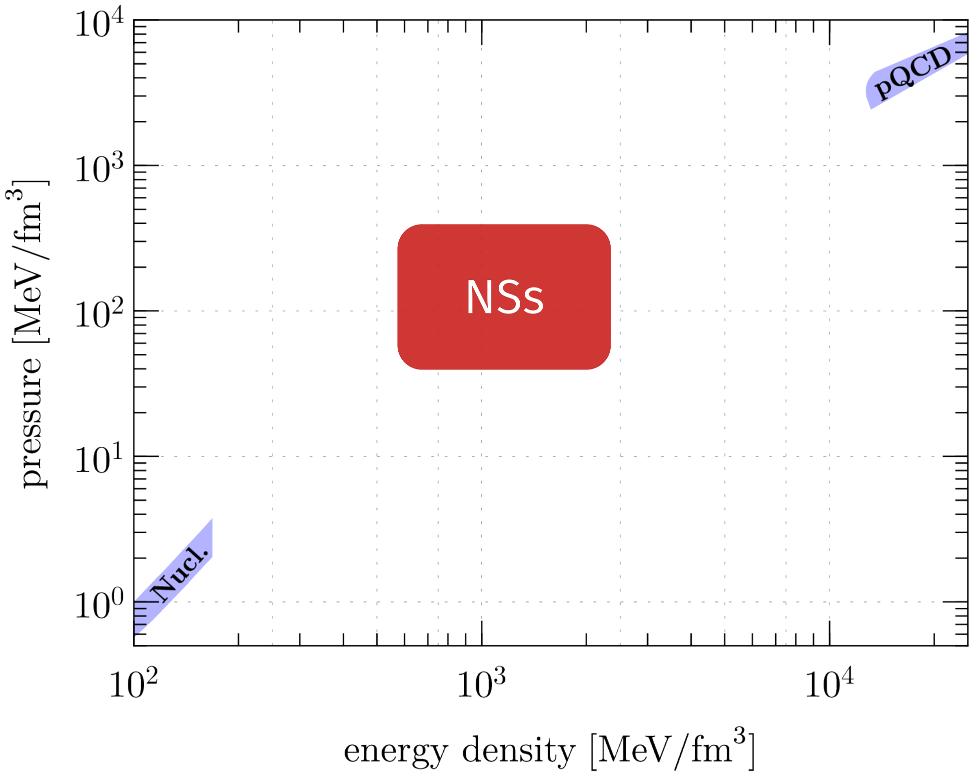
TU Darmstadt

PhD Retreat, Graz (13-15.05.2022)

TECHNISCHE UNIVERSITÄT DARMSTADT

Recap: The EOS of dense matter

NSs probe densities beyond nuclear density, but below pQCD densities



* Last lecture

1. Perform calculations in CET (and pQCD)

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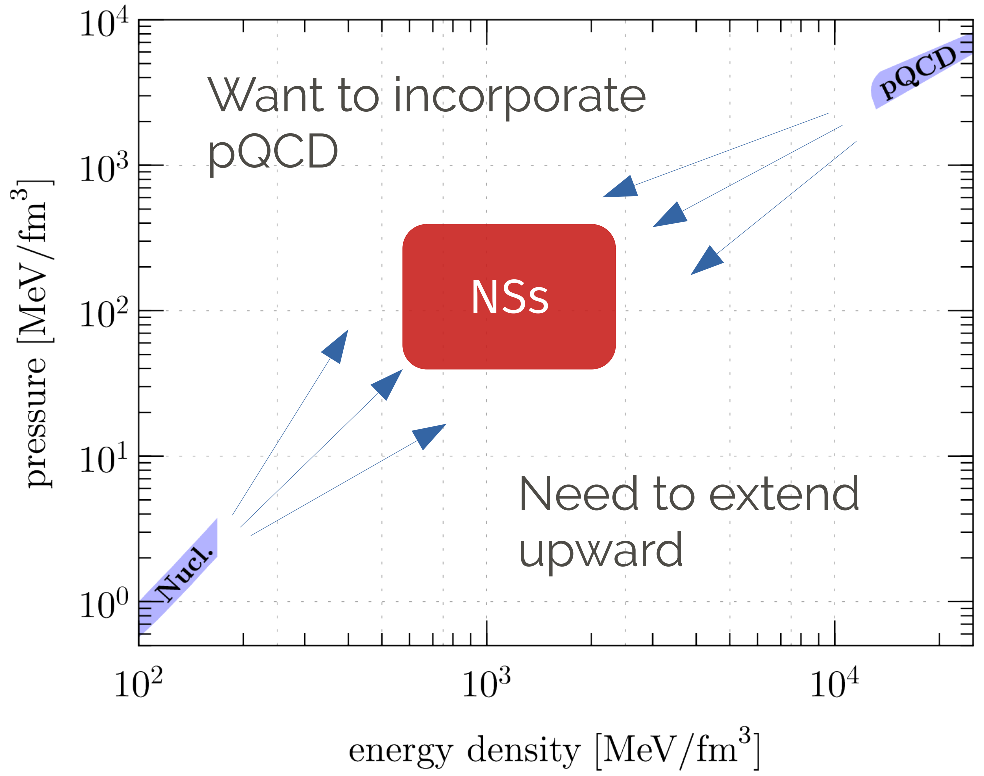

* Last lecture

1. Perform calculations in CET (and pQCD)

2. Extend EOSs to NS regime (ensemble)

3. Fold in NS observations to decrease uncertainties

* This lecture!

Can we constrain the phase of dense matter? (1/2)

- *Quark matter [1] (QM) has different physical properties than hadronic matter [2] (HM):*

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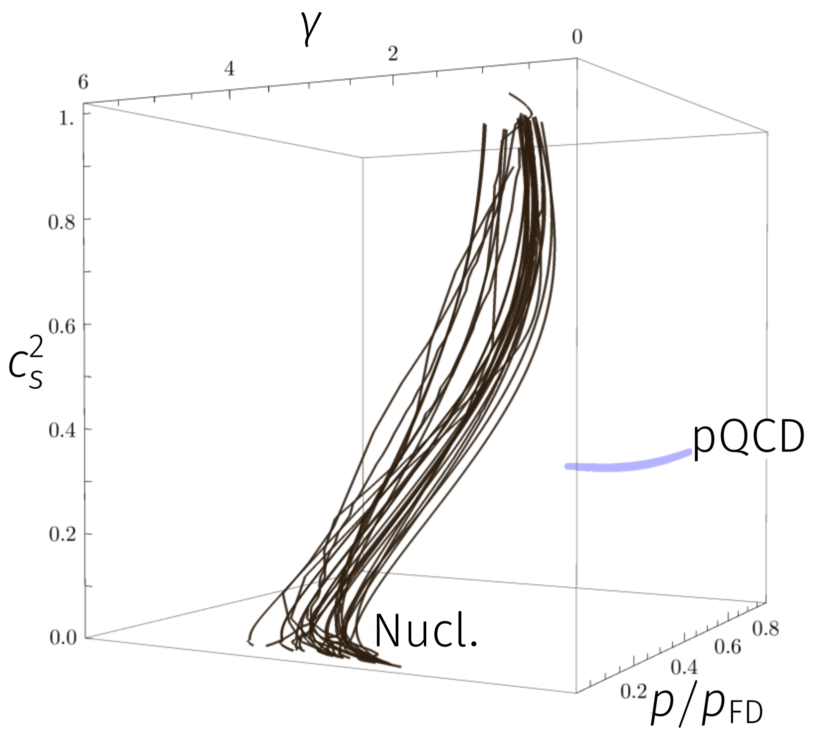
  [2]: Fortin+ Phys. Rev. C 94, (2016), Lattimer & Prakash, Astrophys. J. 550 (2001), Gandolfi+ Phys. Rev. C 85 (2012)

	Hadronic	Quark
c_s^2	increases	$\lesssim 1/3$
$\gamma \equiv \frac{\mathrm{d}\ln p}{\mathrm{d}\ln \varepsilon}$	≈ 2.5	≈ 1
p/p_{FD}	$\approx 0.1 - 0.3$	$\approx 0.5 - 0.8$



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

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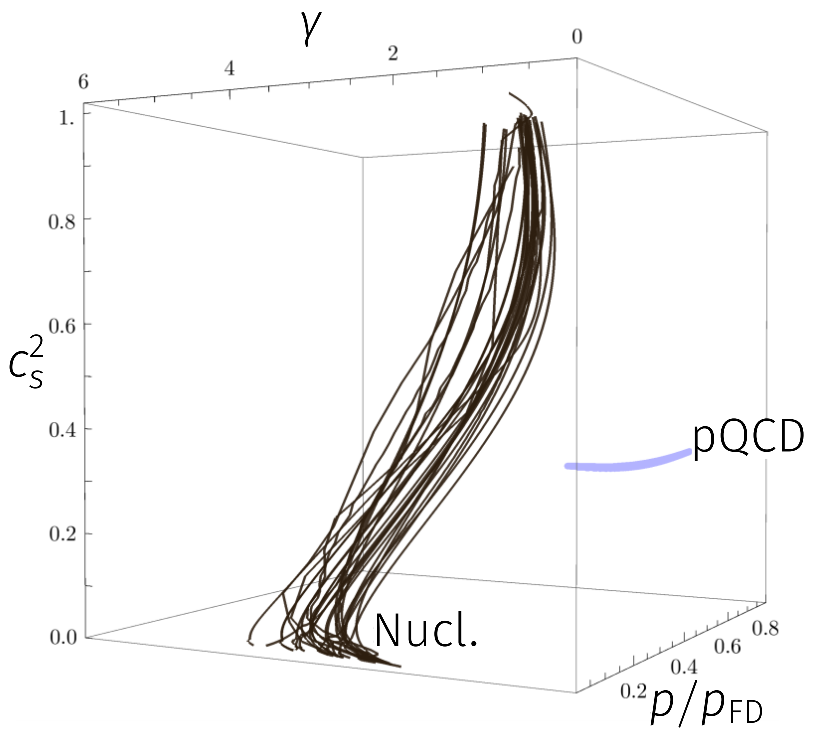
- *Strategy:*

  Identify where EoS changes physical properties from hadronic → quark

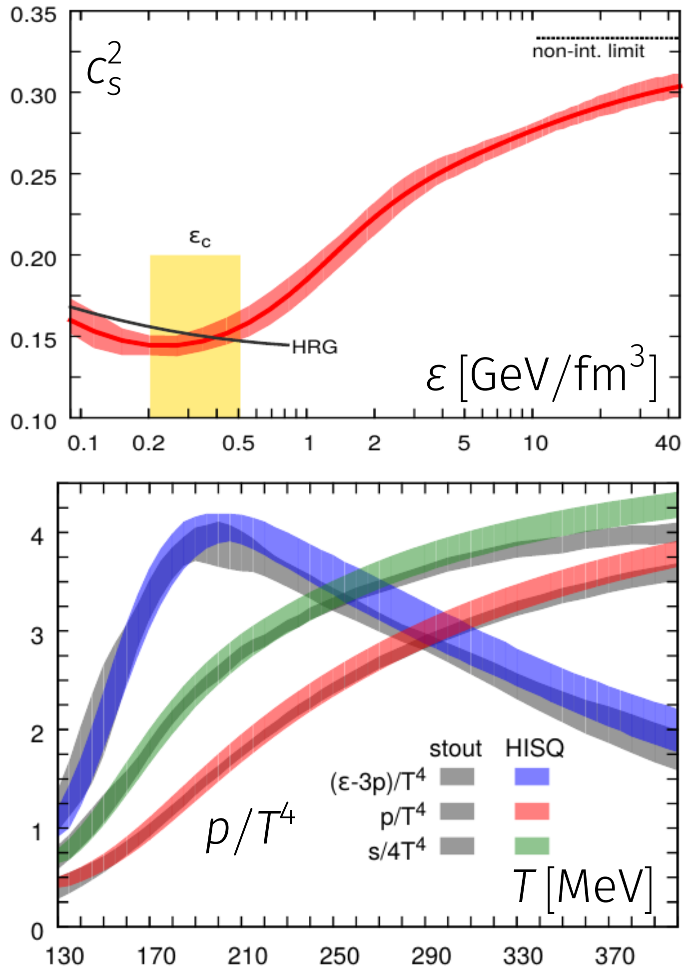


Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

Can we constrain the phase of dense matter? (1/2)

- Similar to looking for change in behavior of lattice results at high *T*.

- Identify change in phase from *change in physical properties* of matter



HotQCD Phys.Rev.D 90 (2014), Borsanyi+ Phys. Lett. B 370 (2014)

Outline

1. Full interpolation from CET to pQCD
2. Apply pQCD at lower densities?
3. Likelihood analysis, studying pQCD impact

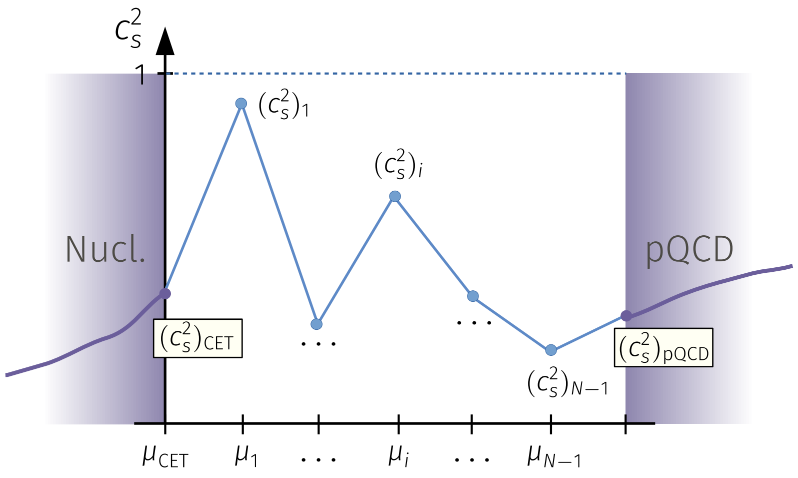
Outline

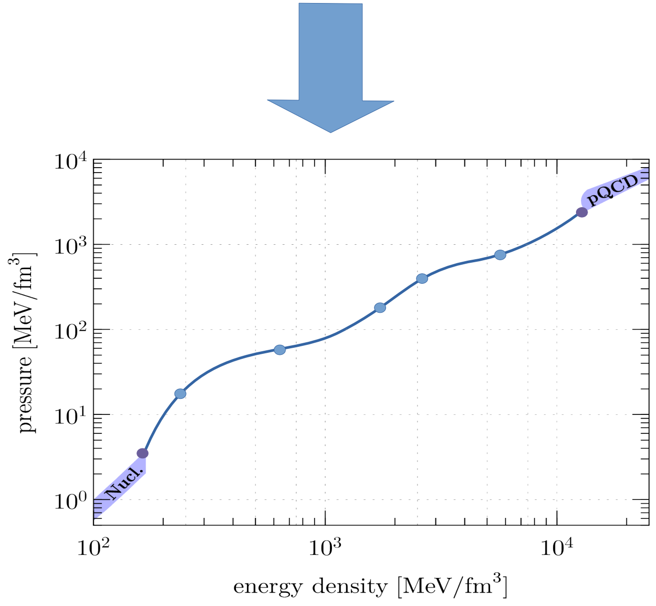
1. **Full interpolation from CET to pQCD**
2. Apply pQCD at lower densities?
3. Likelihood analysis, studying pQCD impact

Full interpolaiton: Constructing an EOS ensemble

- Only theory constraints:
  - CET +pQCD (where valid)
  - $0 \leq c_s^2 < c^2$ (stability + causality)
- *Interpolation:* Sample $\{\mu_i, c^2_{s,i}\}$ points; connect linearly (simple to do)

  Annala, TG, Kurkela, Nättilä, Vuorinen, Nat. Phys. 16 (2020)





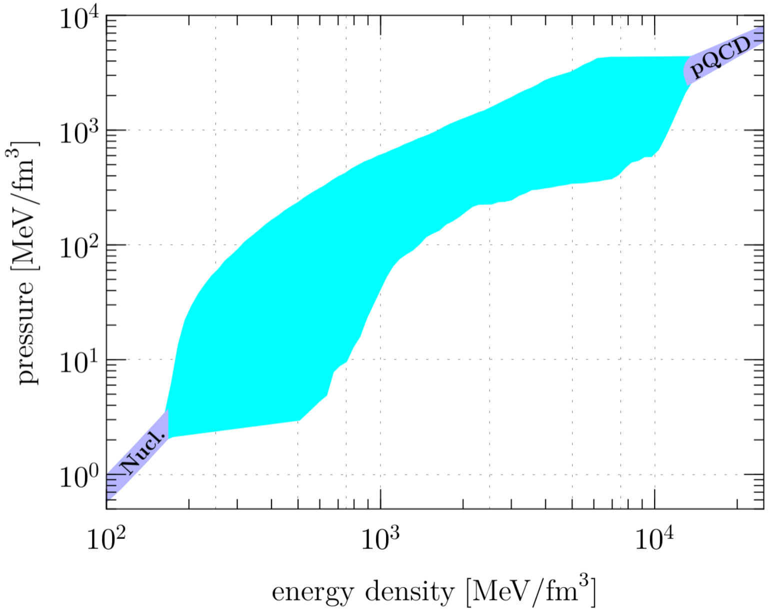
Integrate twice:

$$c_s^2(\mu) = \frac{n}{\mu}\left(\frac{dn}{d\mu}\right)^{-1}, \qquad n = \frac{dp}{d\mu}$$

Full interpolaiton: Constructing an EOS ensemble

- Only theory constraints:
  - CET +pQCD (where valid)
  - $0 \leq c_s^2 < c^2$ (stability + causality)
- *Interpolation:* Sample $\{\mu_i, c_{s,i}^2\}$ points; connect linearly (simple to do)

  Annala, TG, Kurkela, Nättilä, Vuorinen, Nat. Phys. 16 (2020)
- Matching to CET, pQCD in (ε, p, n) sets theory bounds on EoS
- *Can now fold in observations*



Fold in two observations from Lecture 1

1. *High-mass pulsars*

$$M_{\rm TOV} \geq \begin{cases} 1.97 \pm 0.04 M_\odot \\ 2.01 \pm 0.04 M_\odot \\ 2.08 \pm 0.07 M_\odot \end{cases}$$

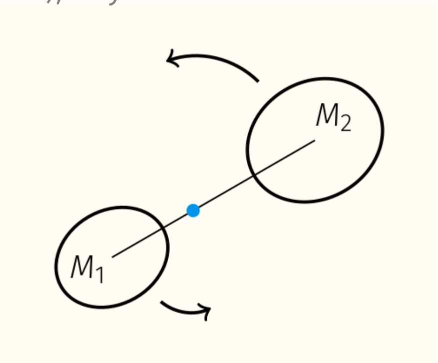
Demorest+ Nature 467 (2010),
Antoniadis+ Science 240 (2013),
Fonseca+ Astrophys. J. Lett. 915 (2021)

2. *GW170817*

$$\tilde\Lambda < 720, \text{ with } \mathcal{M}_{\rm chirp} = 1.186 M_\odot,$$
$$q \equiv M_2/M_1 \in [0.7, 1]$$

Abbott+ Phys. Rev. Lett. 119 (2017); Phys. Rev. Lett. 121 (2018); Phys. Rev. X 9 (2019).

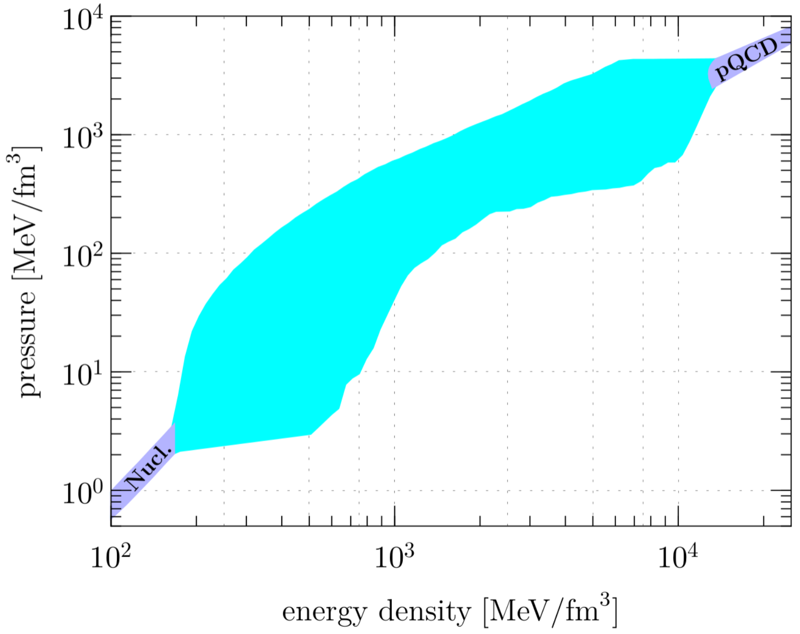
$$\Lambda(M) \equiv |Q_{ij}/\mathcal{E}_{ij}|M^5$$



$$\tilde\Lambda \equiv \frac{16}{13}\left[\frac{(M_1 + 12M_2)M_1^4}{(M_1+M_2)^5}\Lambda(M_1) + (1 \leftrightarrow 2)\right];$$

$$\mathcal{M}_{\rm chirp} \equiv \frac{(M_1 M_2)^{3/5}}{(M_1+M_2)^{1/5}}$$

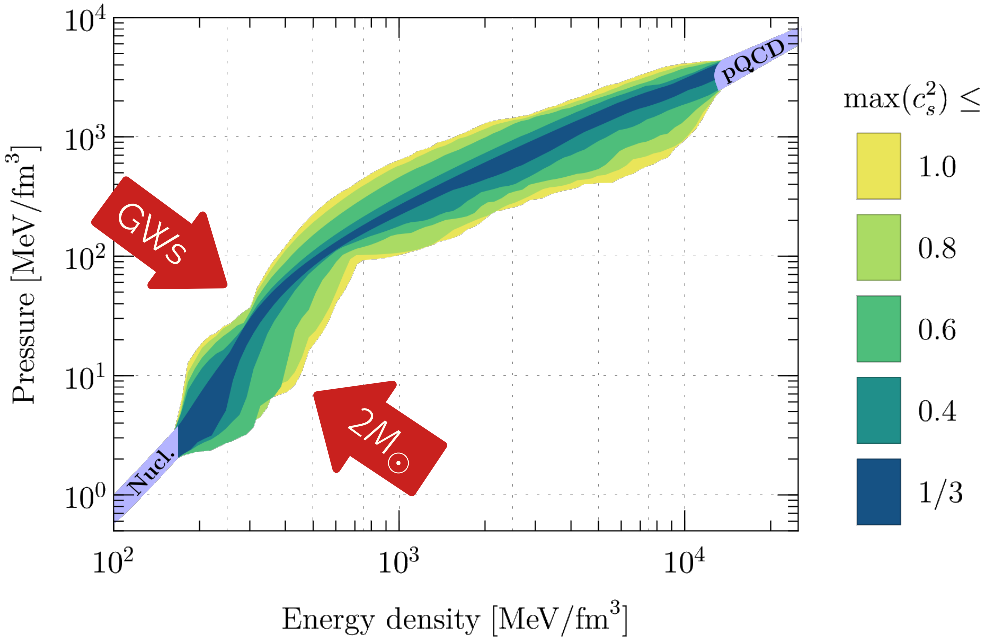
Implementing hard cuts shows change in behavior



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

Implementing hard cuts shows change in behavior

- M and Λ constraints complementary constrain at low densities



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

Implementing hard cuts shows change in behavior

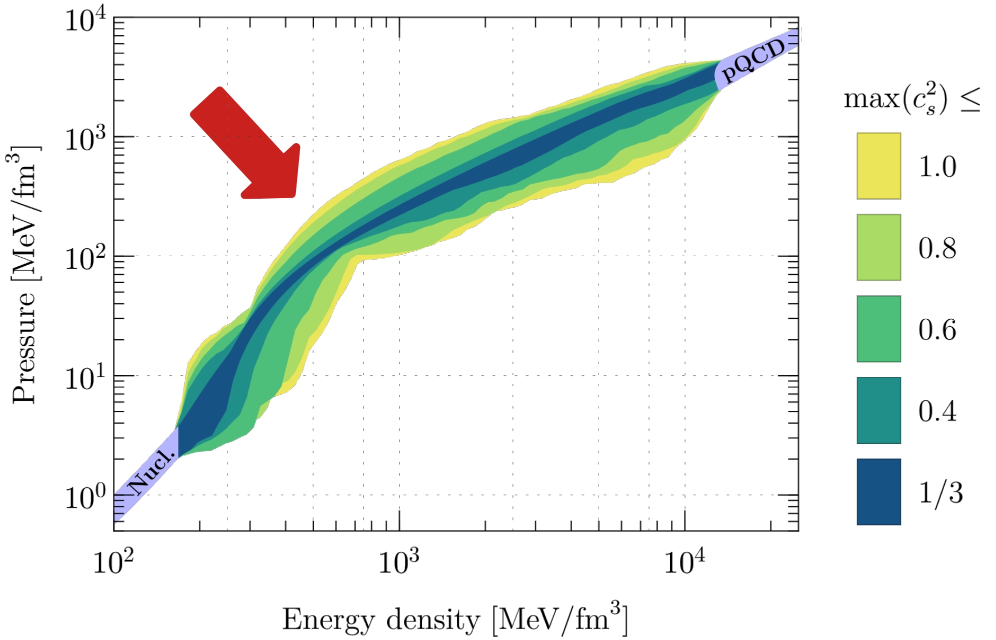
- M and Λ constraints complementary constrain at low densities

- See bend in EoS band:
  - Nonconformal→conformal

$$\gamma \equiv \frac{\mathrm{d}\ln p}{\mathrm{d}\ln \varepsilon}; \gamma \approx 2.5 \mapsto \gamma \approx 1$$

  - Location near crossover transition at high T

HotQCD: Phys. Rev. D 90 (2014)



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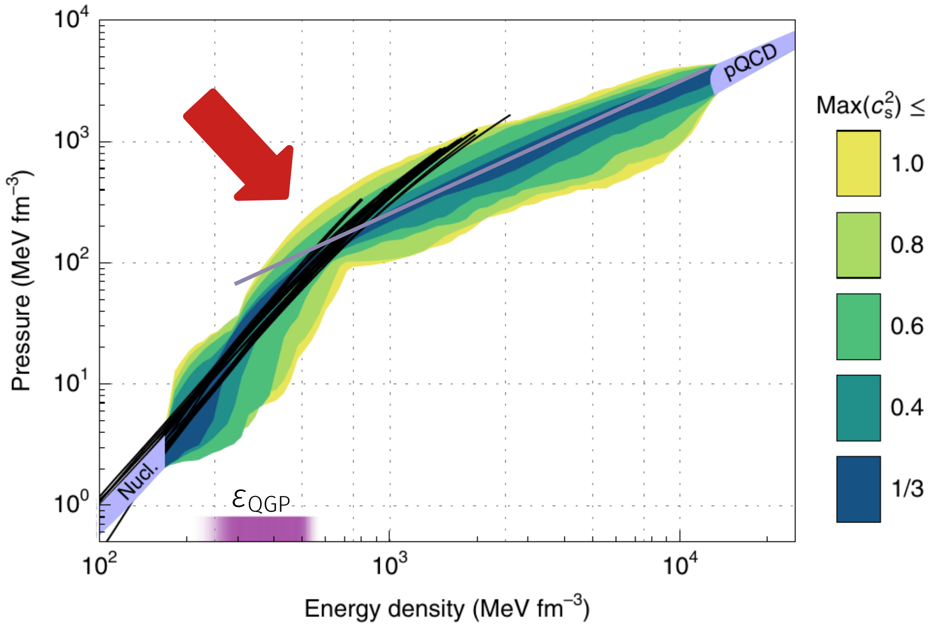

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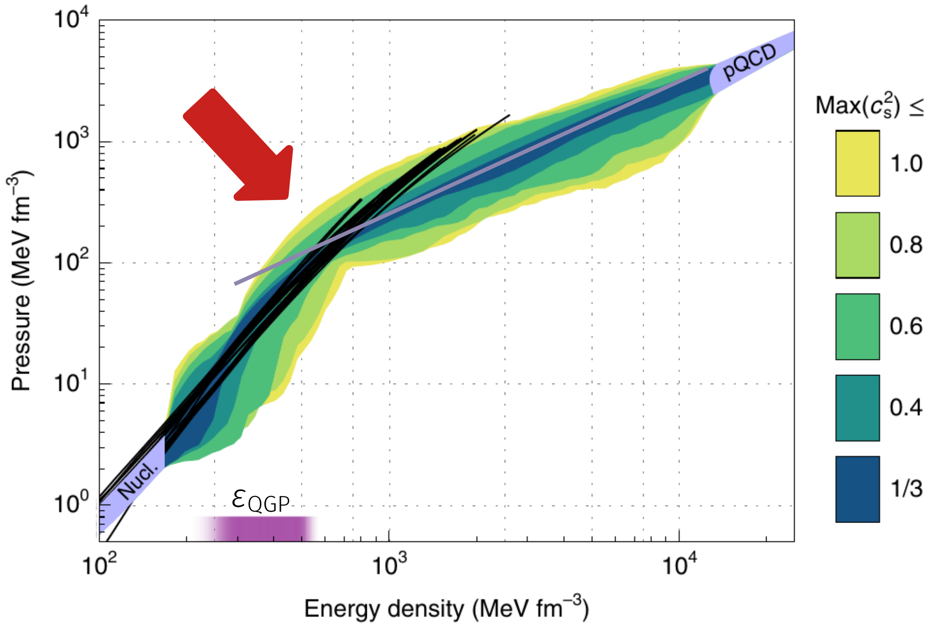
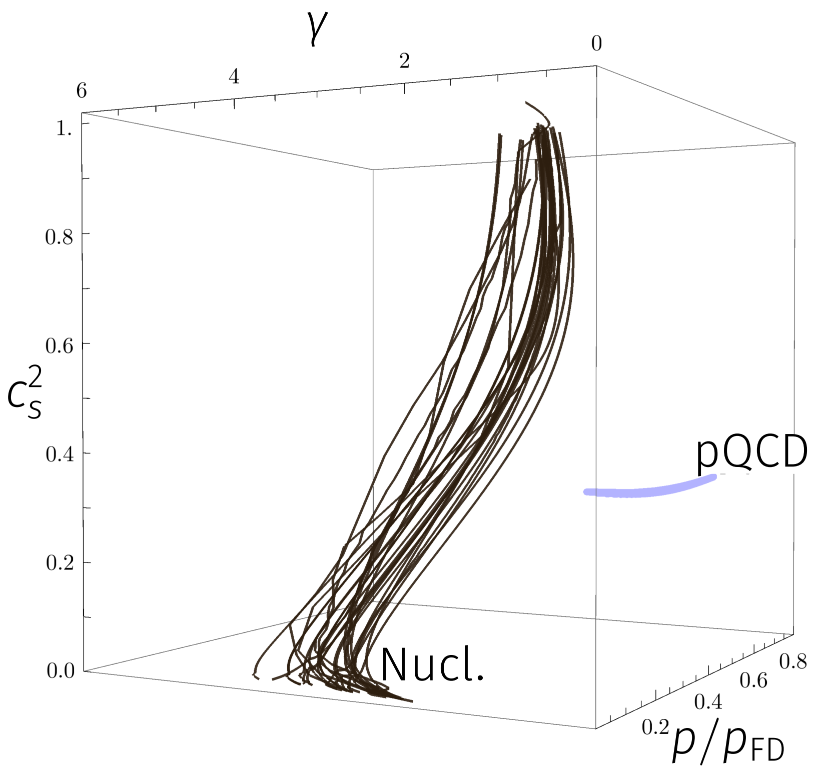
    HotQCD: Phys. Rev. D 90 (2014)

- *Suggestive; but need to investigate on EoS-by-EoS basis*



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

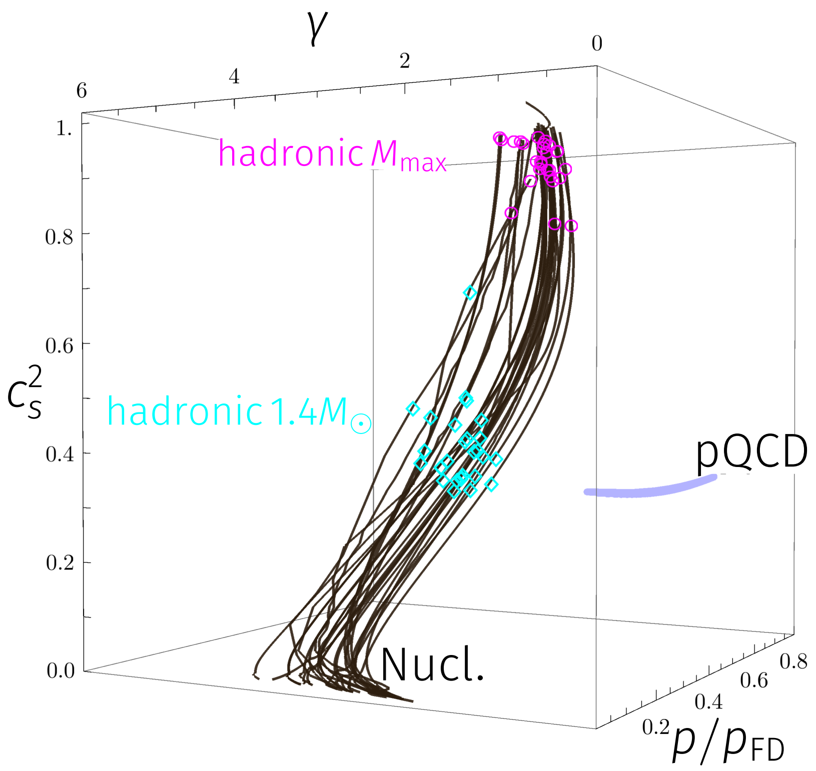
Evidence for QM cores



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

Evidence for QM cores

- Centers of $1.4M_\odot$, $M_{\max}$, stars for nucl. models



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

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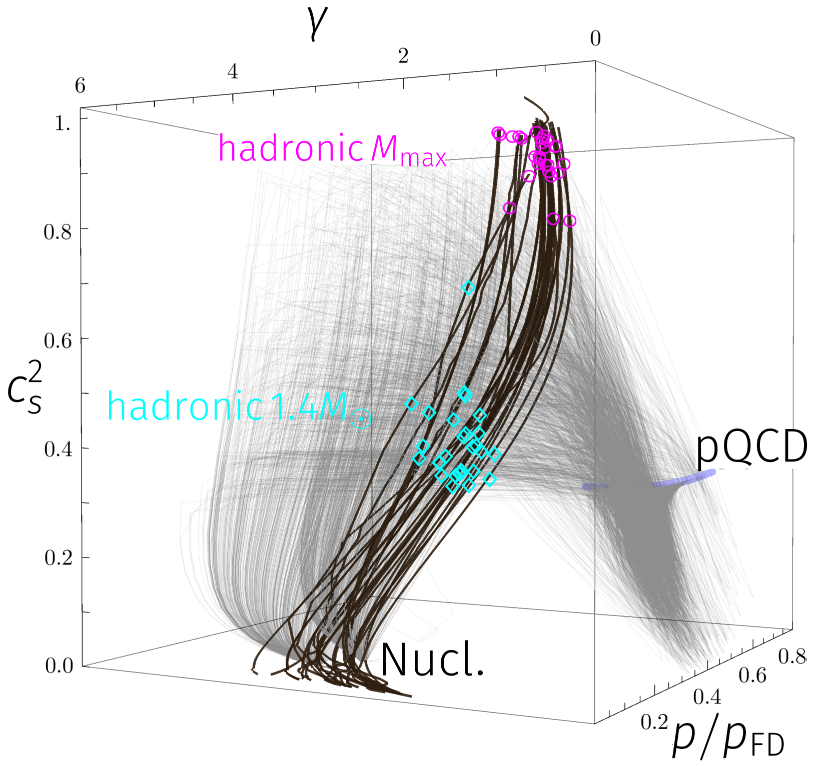

Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

Evidence for QM cores

- Centers of $1.4M_\odot$, $M_{\max}$, stars for nucl. models
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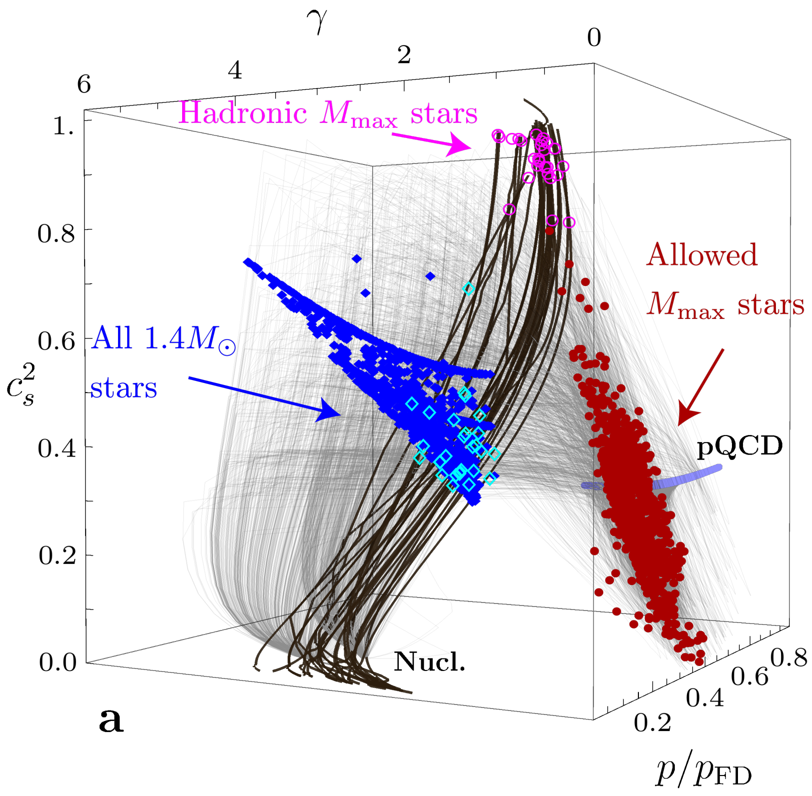
- $1.4M_\odot$ stars ***consistent*** with cores of $1.4M_\odot$ Nucl.
- (most) $M_{\max}$ stars ***inconsistent*** with centers of Nucl. $M_{\max}$ ($\max(c_s^2) < 0.7$)



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

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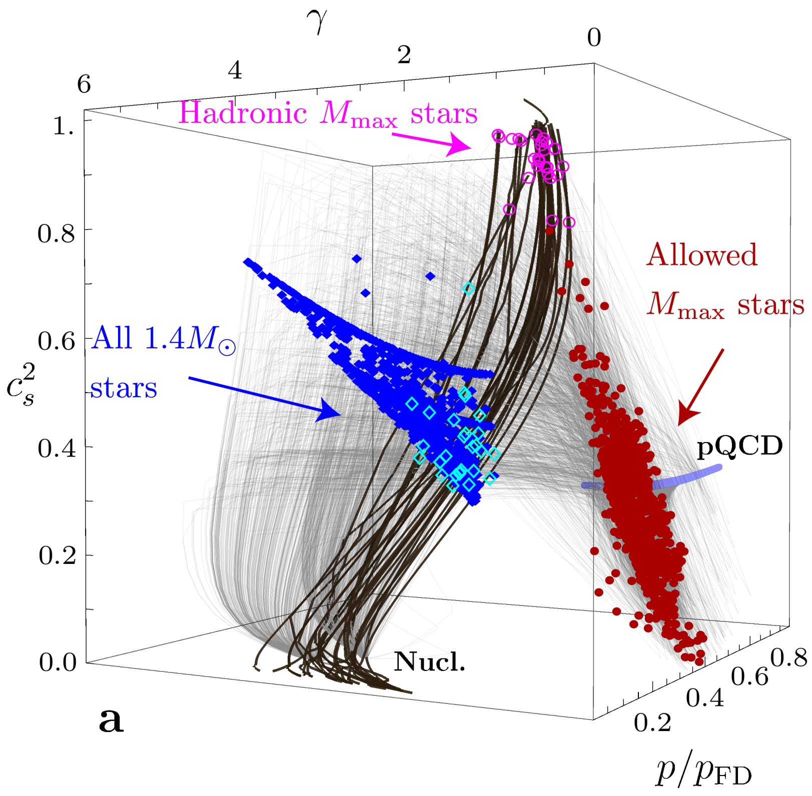
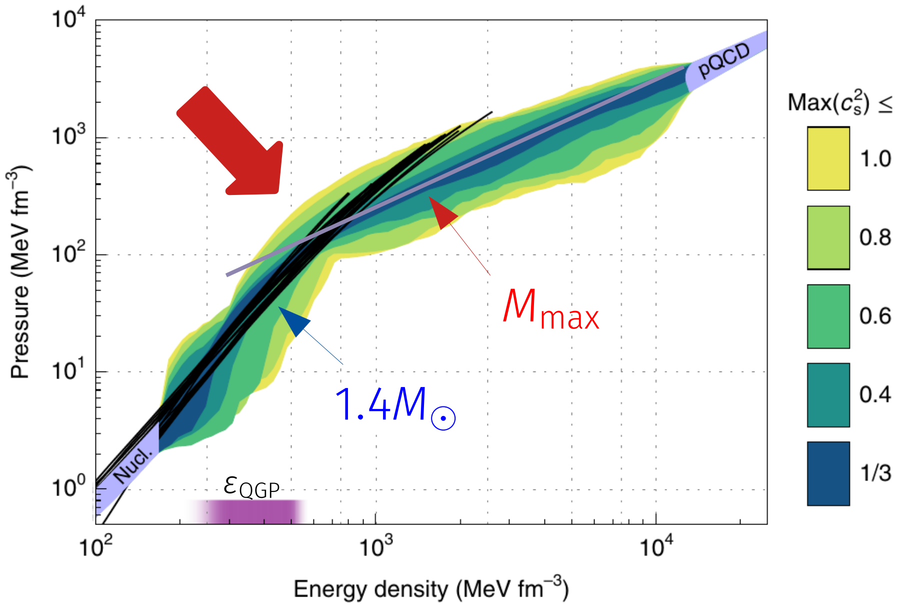
- (most) $M_{\max}$ stars *inconsistent* with centers of Nucl. $M_{\max}$ ($\max(c_s^2) < 0.7$)
- Properties of EoS *remain closer QM to asymptotic densities*



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

Evidence for QM cores: *Takeaways*

- Nucl. → Quark thermodynamic transition for *vast majority* of the ensemble



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

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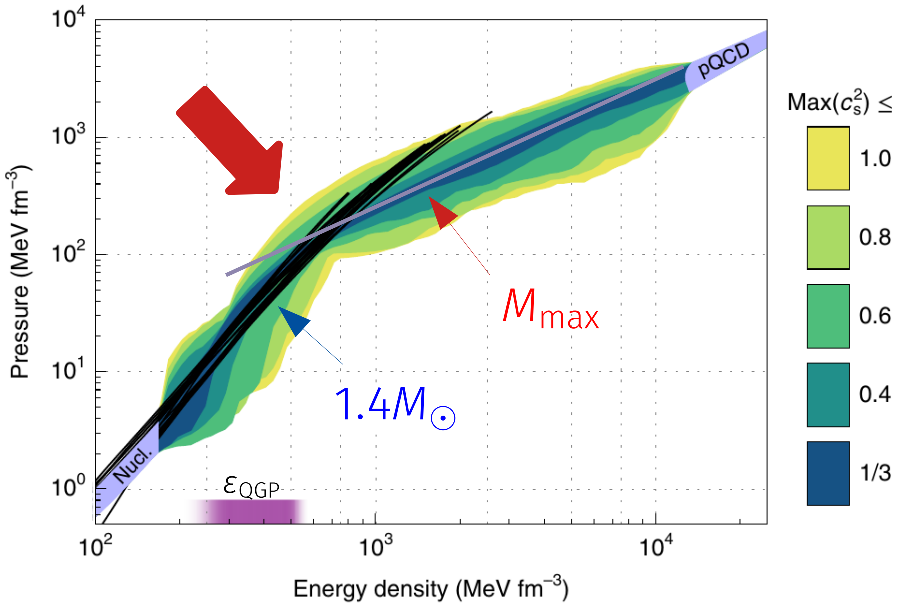

Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

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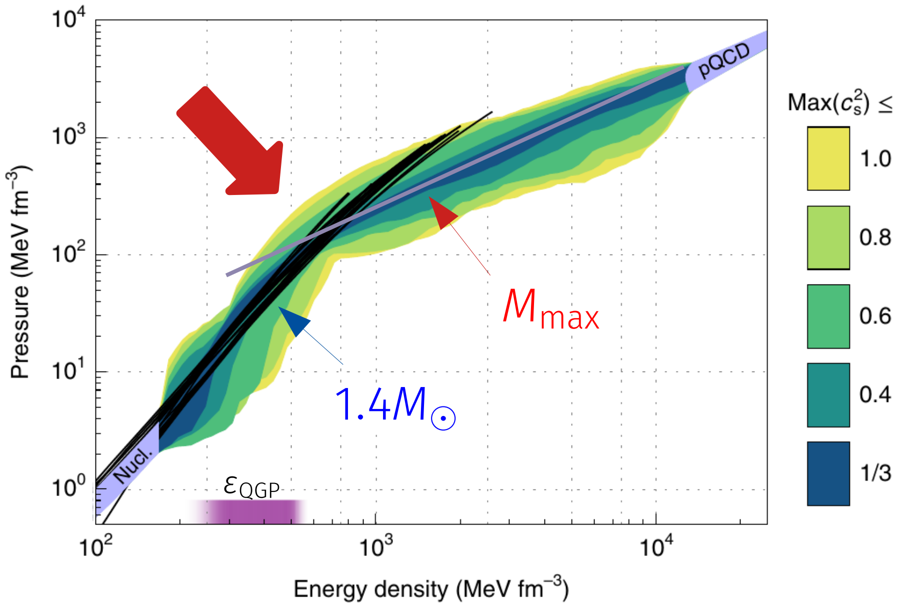
- For core to be absent, need:

    1. PT with $\Delta\varepsilon > 130\ \mathrm{MeV/fm}^3$, $\Delta\varepsilon/\varepsilon > 0.2$

    2. AND $\max(c_s^2) > 0.7c^2$



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

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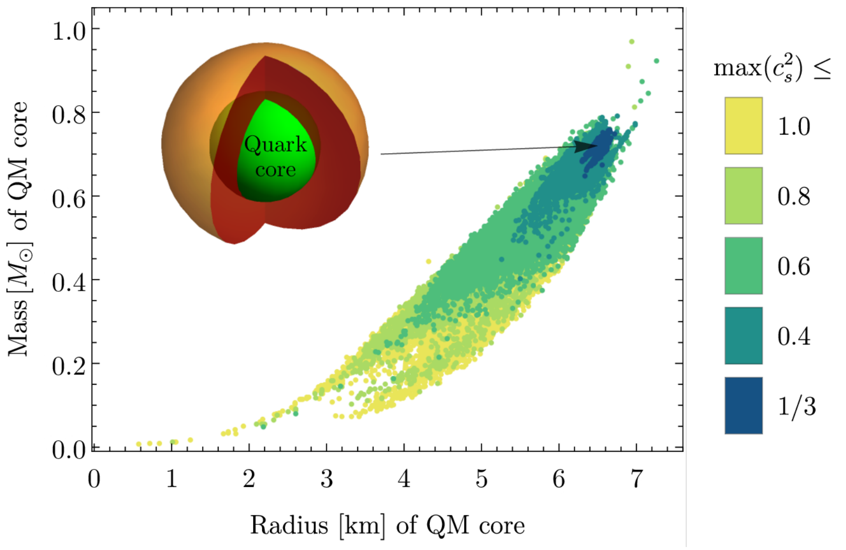
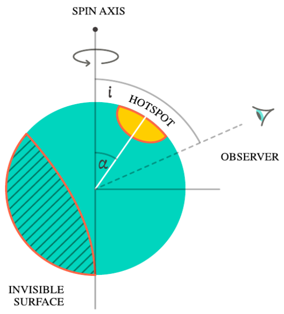
  1. PT with $\Delta\varepsilon > 130$ MeV/fm^3, $\Delta\varepsilon/\varepsilon > 0.2$
  2. AND $\max(c_s^2) > 0.7c^2$

- Sizeable cores, if conformal bound not strongly broken ($\max(c_s^2) < 0.5c^2$)

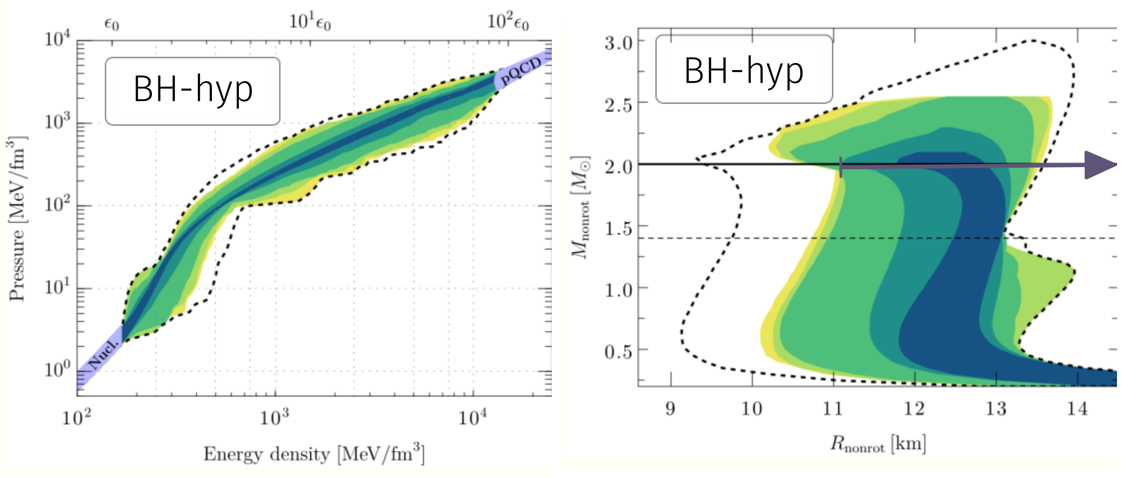


Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

Evidence for QM cores: *Further constraints*



- NICER $R(2M_\odot) \geq 11.0$ km
  + BH-hyp in GW170817



Annala, TG, Katerini, Kurkela, Nättilä, Paschalidis, Vuorinen
Phys.Rev.X 12 (2022)

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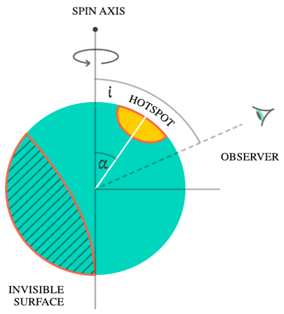
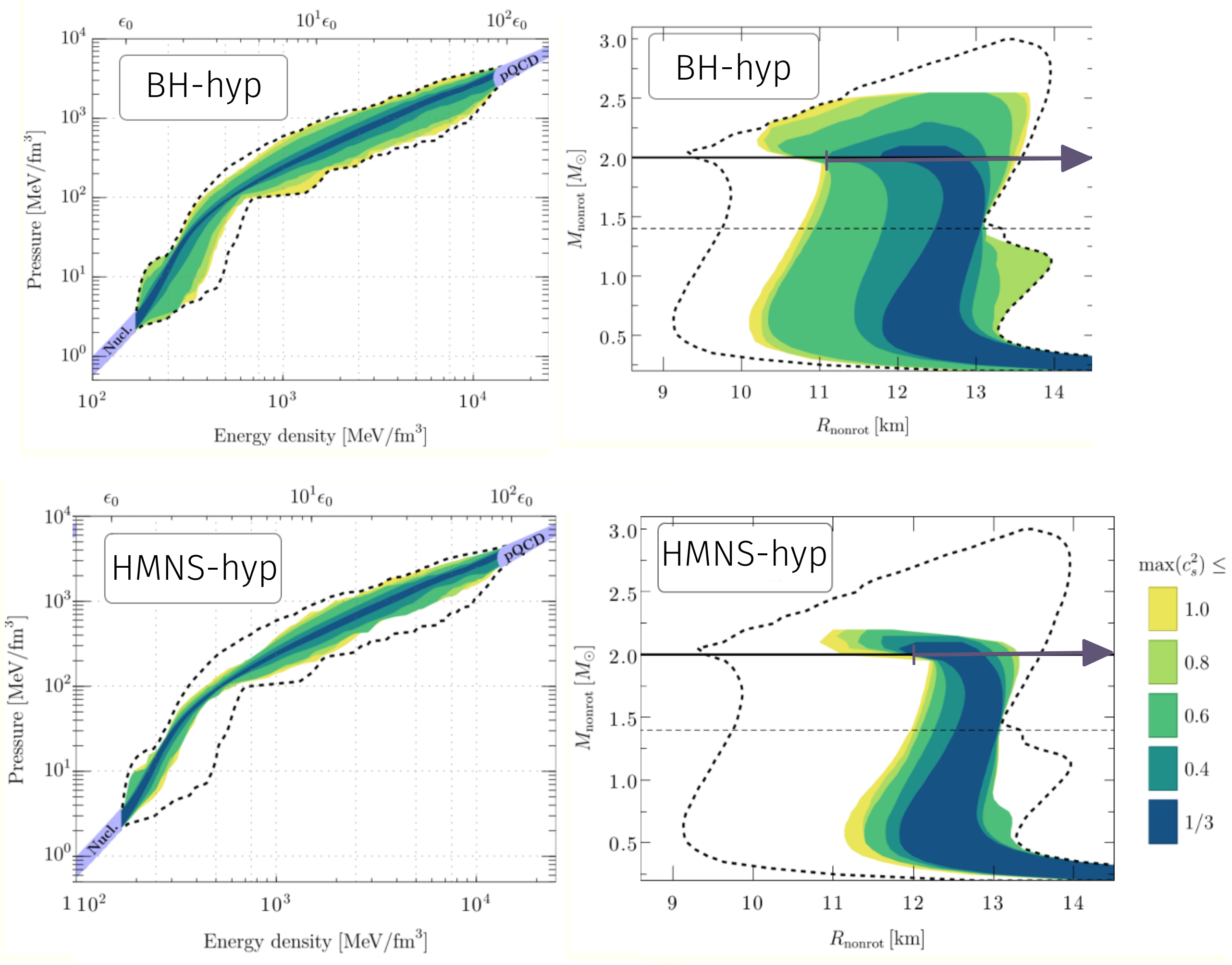


- NICER $R(2M_\odot) \geq 11.0$ km + BH-hyp in GW170817

- ***Most restrictive primarily removes EoSs without QM cores***

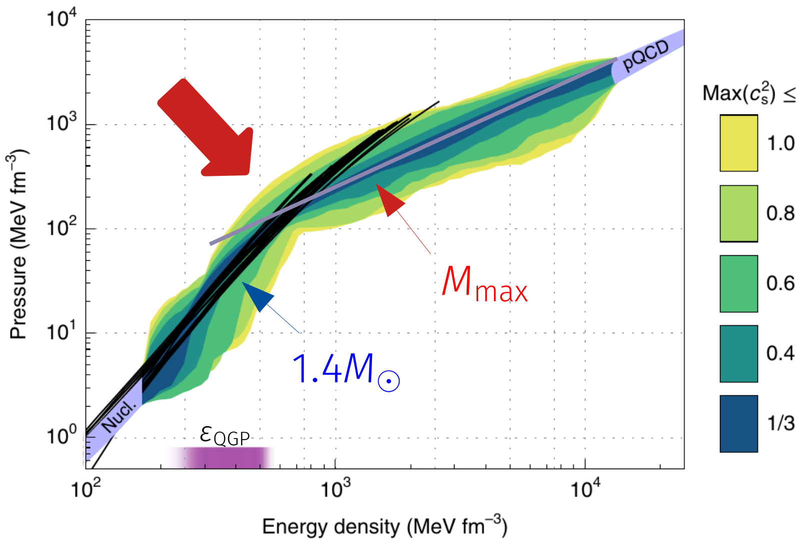
  $R(2M_\odot) \geq 12.2$ km
  +hypermassive NS in GW170817



Annala, TG, Katerini, Kurkela, Nättilä, Paschalidis, Vuorinen
Phys.Rev.X 12 (2022)

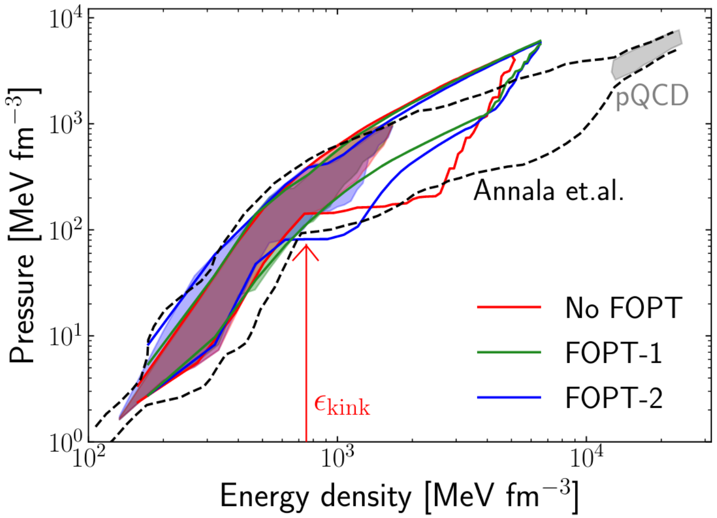
Differences in the literature

Previous works with pQCD constraint see some softening transition along physical NS sequence, while other works without it do not



Visual summary of above

VS.



Somasundaram, Tews, Margueron
2112.08157

Differences in the literature

Previous works with pQCD constraint see some softening transition along physical NS sequence, while other works without it do not

Question:	*Is softening a genuine (p)QCD prediction, or a result of interpolation through 2 orders of magnitude in density?*
Past weakness:	*Our past work has all been with hard cuts & not full measurement uncertainties*

Outline

1. Full interpolation from CET to pQCD
2. **Apply pQCD at lower densities?**
3. Likelihood analysis, studying pQCD impact

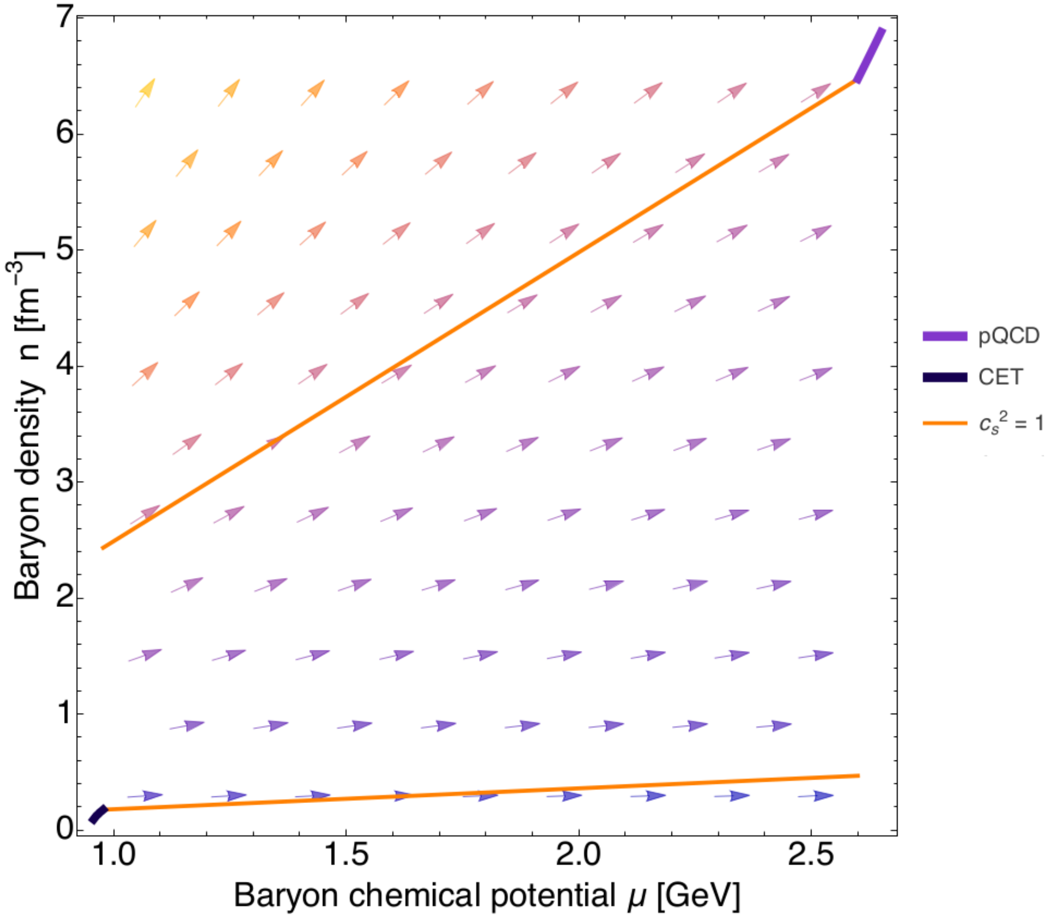
How to feed down QCD input to lower densities

Komoltsev and Kurkela , arXiv:2111.05350

1. Stability

2. Causality

3. Consistency



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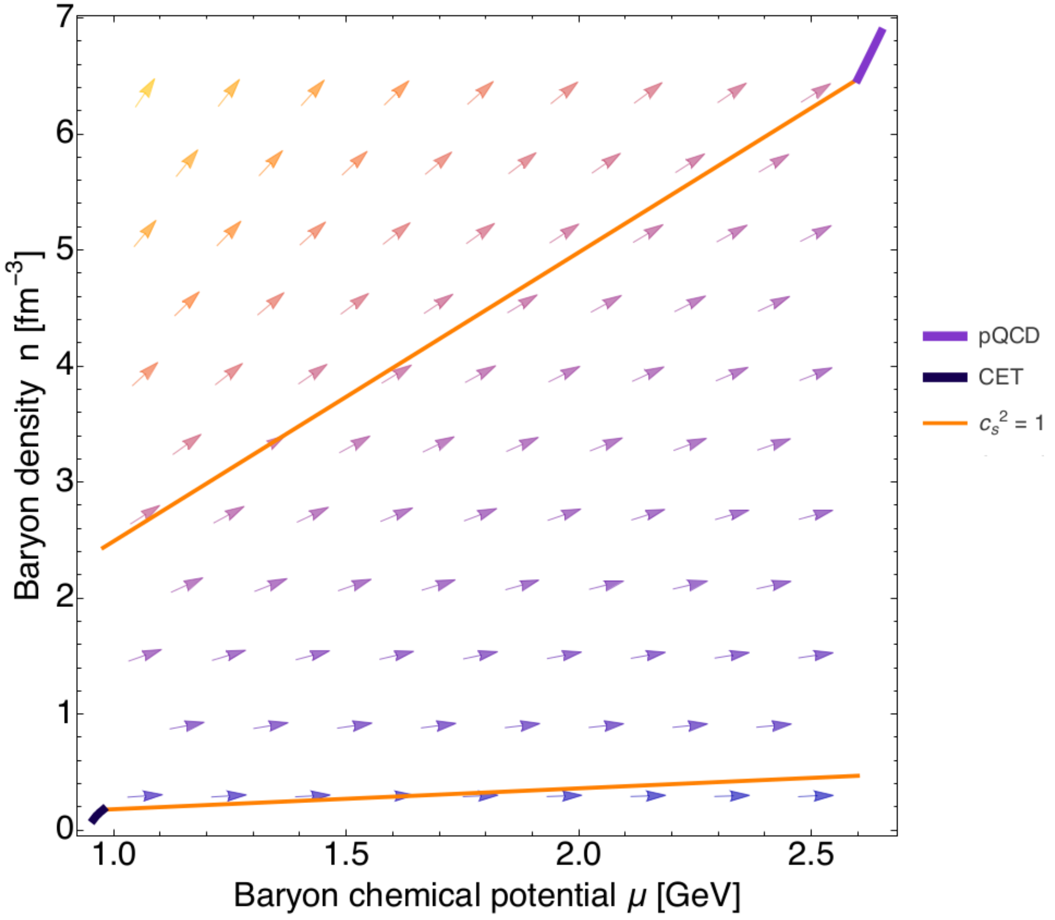

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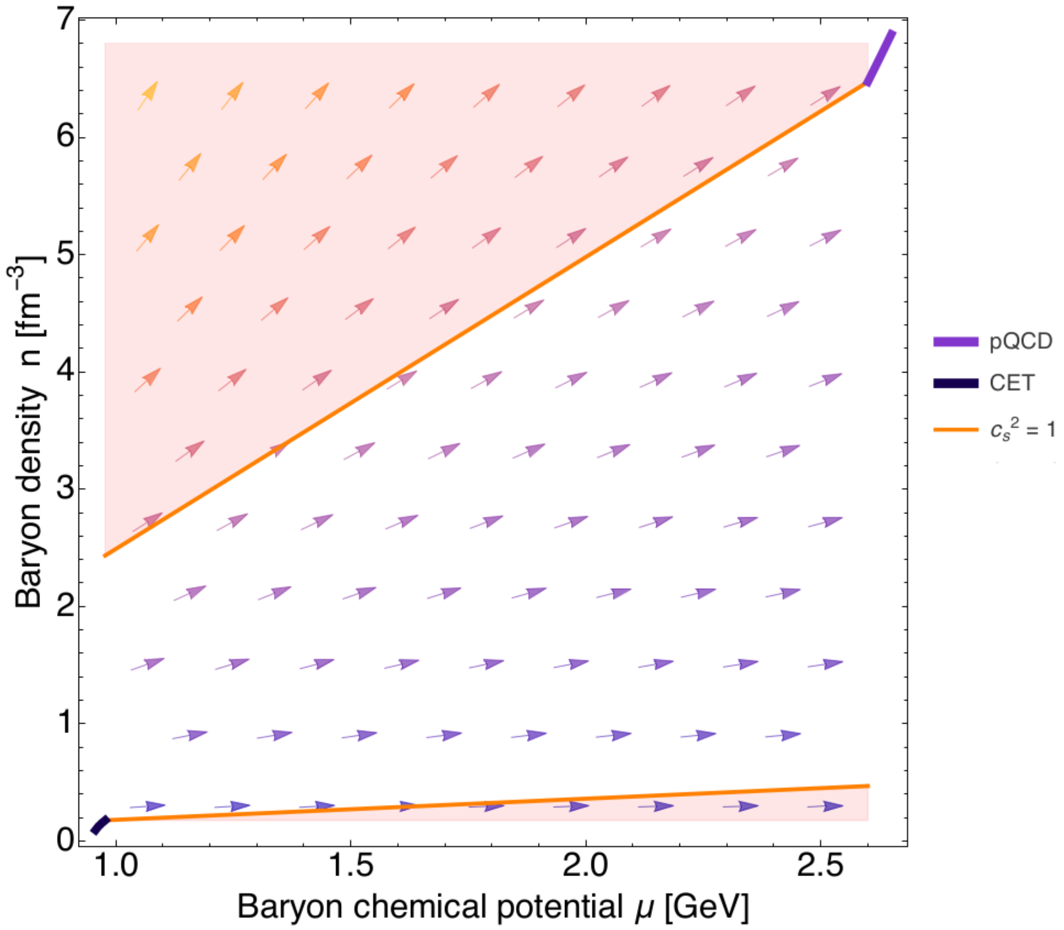

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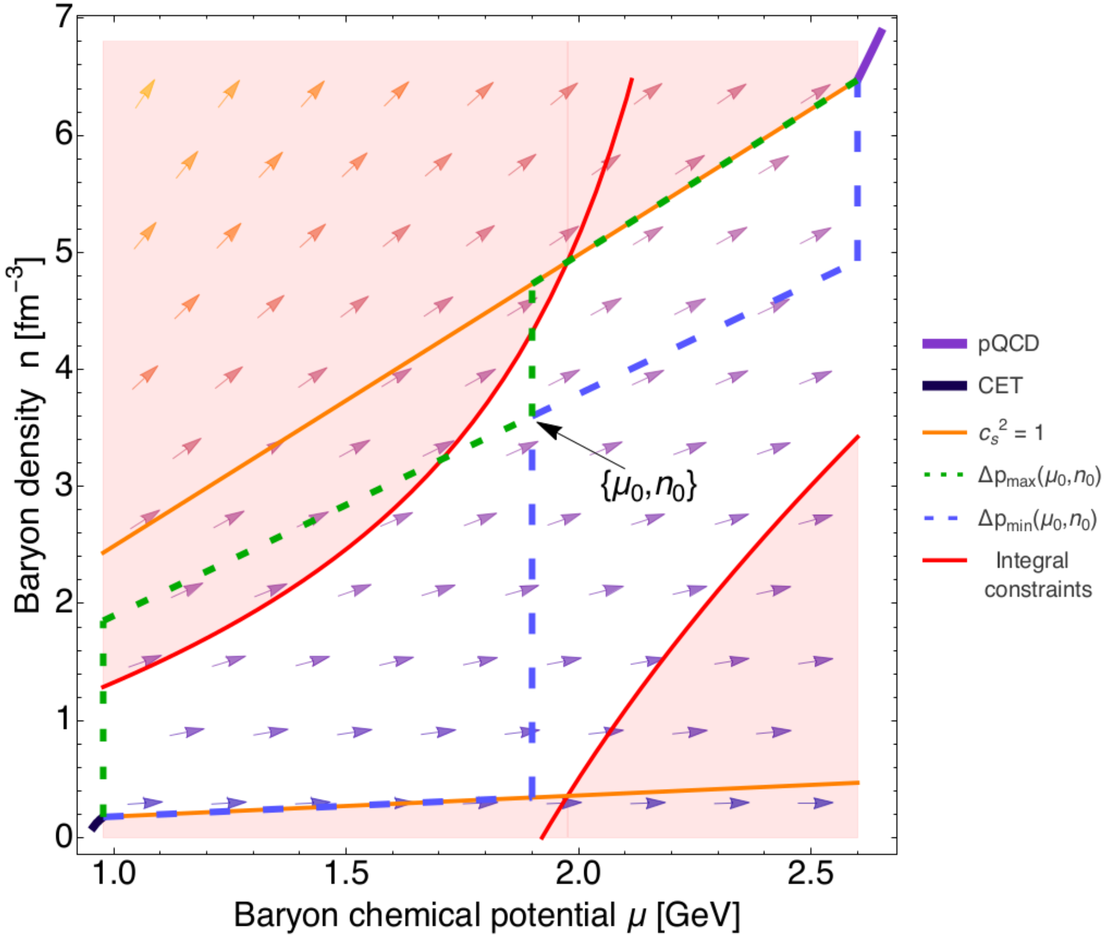
3. Consistency

$$\int_{\mu_{\mathrm{CET}}}^{\mu_{\mathrm{QCD}}} \mathrm{d}\mu\, n(\mu) = p_{\mathrm{QCD}} - p_{\mathrm{CET}}$$

Fixed!



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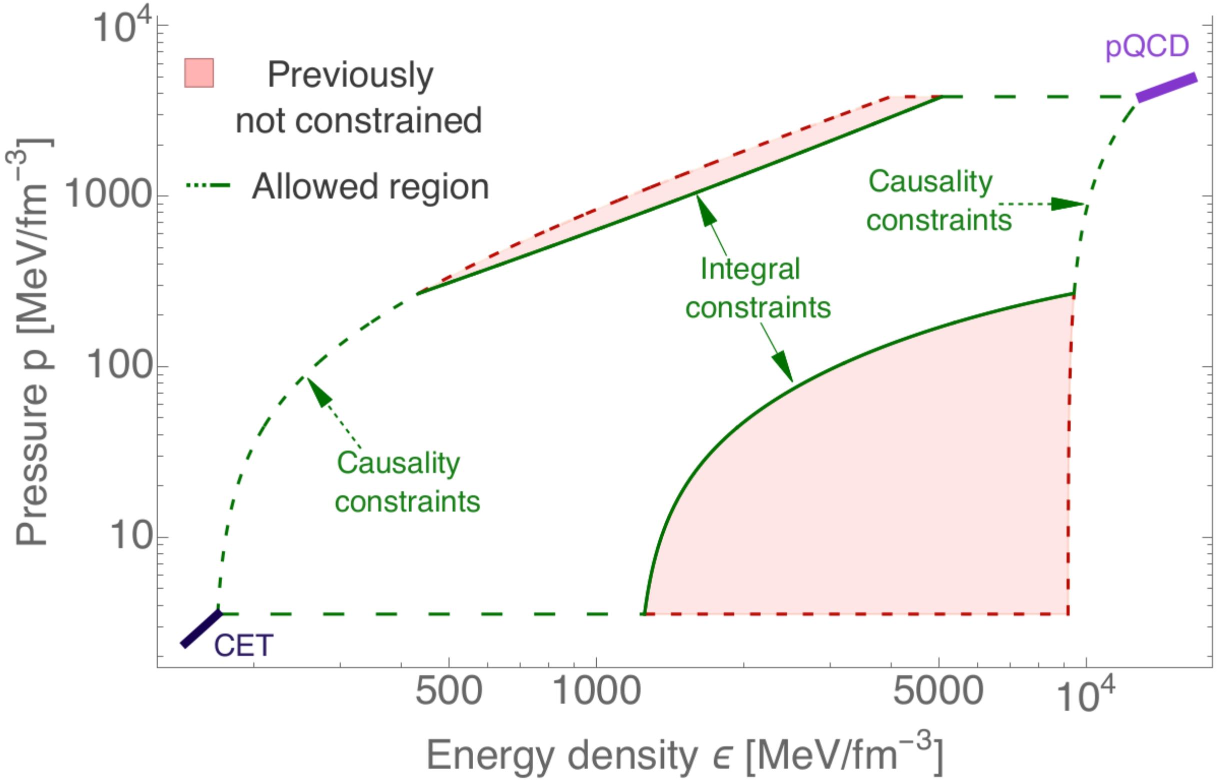

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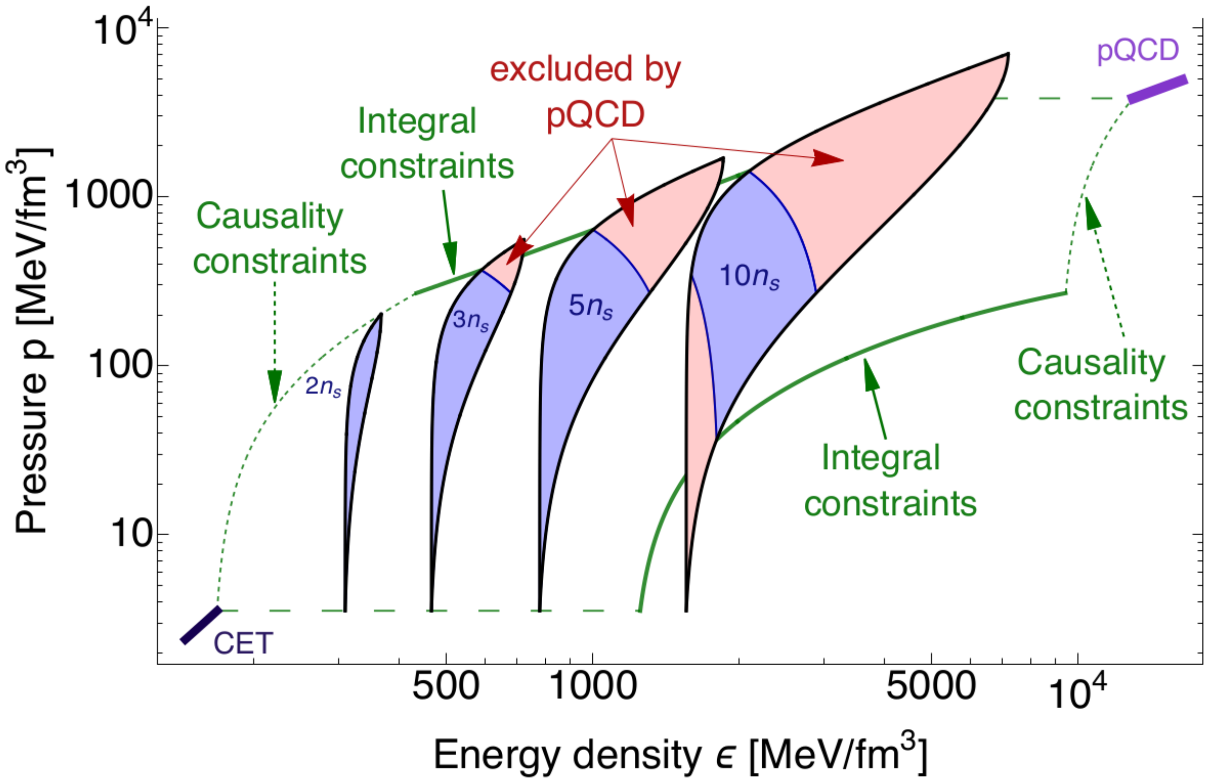

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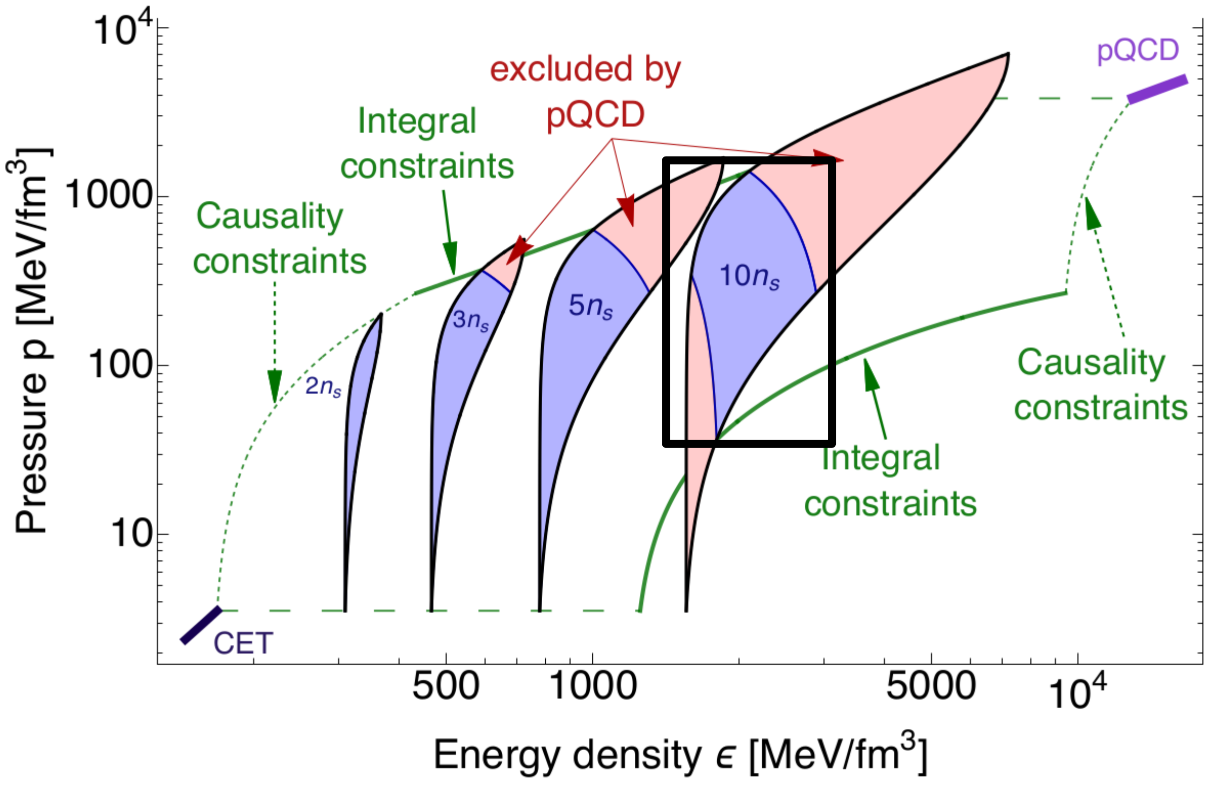


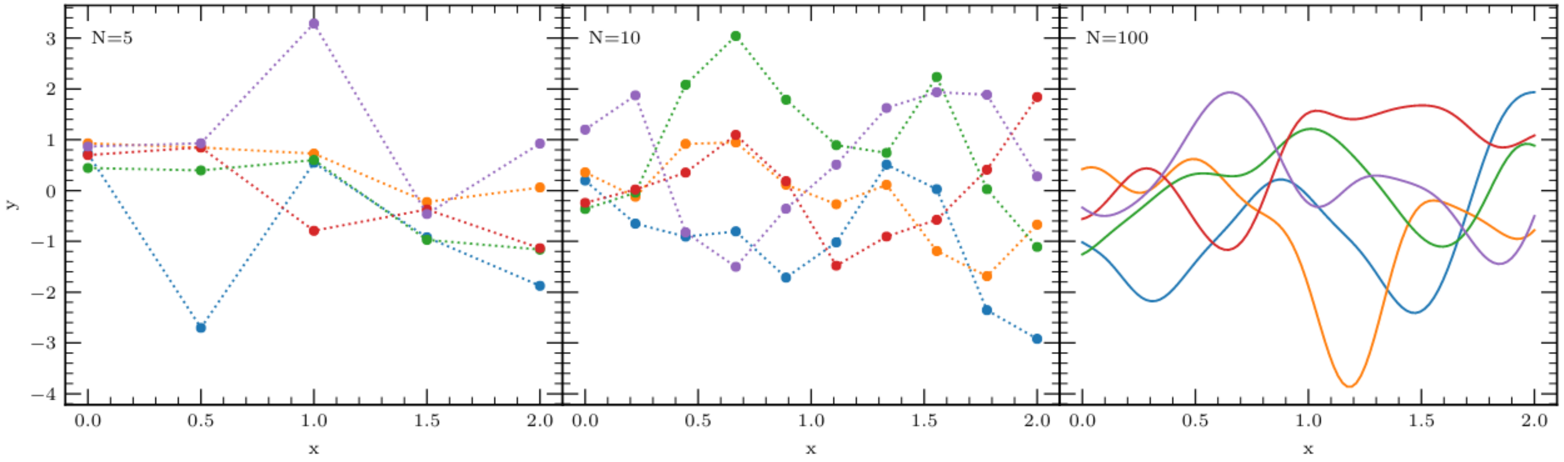
Want to use this n = 10n_s region as high-density constraint

Outline

1. Full interpolation from CET to pQCD
2. Apply pQCD at lower densities?
3. **Likelihood analysis, studying pQCD impact**

Gaussian Processes: Quick overview 1/2

- Consider random variables $\{Z(x_i), i = 1, 2, \dots n\}$, following a multivariate Gaussian distribution
- Also assume that points with closer x_i values are more tightly correlated
- Then as $n \to \infty$ will get a "Gaussian Process" (random function with Gaussian correlations)
- Write $Z \sim GP(\mu, k)$ with mean $\mu(x_i)$ and covariance $k(x, y)$



Adapted from Jonas Keller

Gaussian Processes: Quick overview 2/2

- Now take Z~GP(μ, k) and fold in some (fixed) data D={x_i, y_i}$_i$

$$Z(x_1), \ldots, Z(x_n), Z(x_1^*), \ldots, Z(x_n^*) \sim \mathcal{N}(\vec{\mu}, \Sigma)$$

- Posterior distribution for remaining points is still a Gaussian (think of plugging in points)

$$Z(x_1^*), \ldots, Z(x_n^*) \sim \mathcal{N}(\vec{\mu}^*, \Sigma^*) \qquad \vec{\mu} = \text{prediction}, \Sigma_{i,i} = \text{ uncertainties}$$

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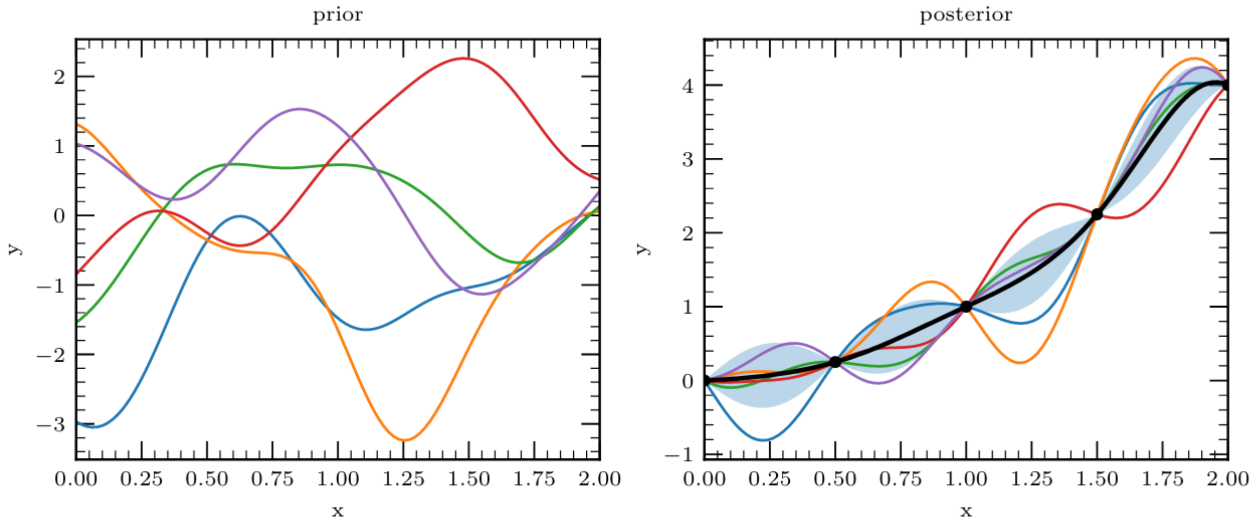
- This way, we can create general functions, fit to some data points



Adapted from Jonas Keller

Setup for Likelihood analysis

TG, Komoltsev, Kurkela, 2204.11877

- Use **Gaussian-Process regression** in auxiliary variable $\varphi(n) = -\ln\left(c_s^{-2}(n) - 1\right)$ to extend CET EOS to $10n_s$

  Similar to Landry & Essick Phys. Rev. D 99 (2019), but for function of *n* instead of *ε*

- *Condition* with low-density CET EOS

  95% CI matching spread of Hebeler, Lattimer, Pethick, Schwenk Astrophys. J. 773 (2013),

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- *Condition* with low-density CET EOS

  95% CI matching spread of Hebeler, Lattimer, Pethick, Schwenk Astrophys. J. 773 (2013),

- Use hierarchical model, with:

$$\varphi(n) \sim \mathcal{N}\left(-\ln\left(\bar{c}_s^{-2} - 1\right), K(n, n')\right), \; K(n, n') = \eta e^{-(n-n')^2/2l^2}$$

- With the hyperparameters themselves drawn from Gaussian distributions:

$$\bar{c}_s^2 \sim \mathcal{N}(0.5, 0.25^2), \; l \sim \mathcal{N}\left(1.0n_s, (0.25n_s)^2\right), \; \eta \sim \mathcal{N}(1.25, 0.25^2).$$

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TG, Komoltsev, Kurkela, 2204.11877

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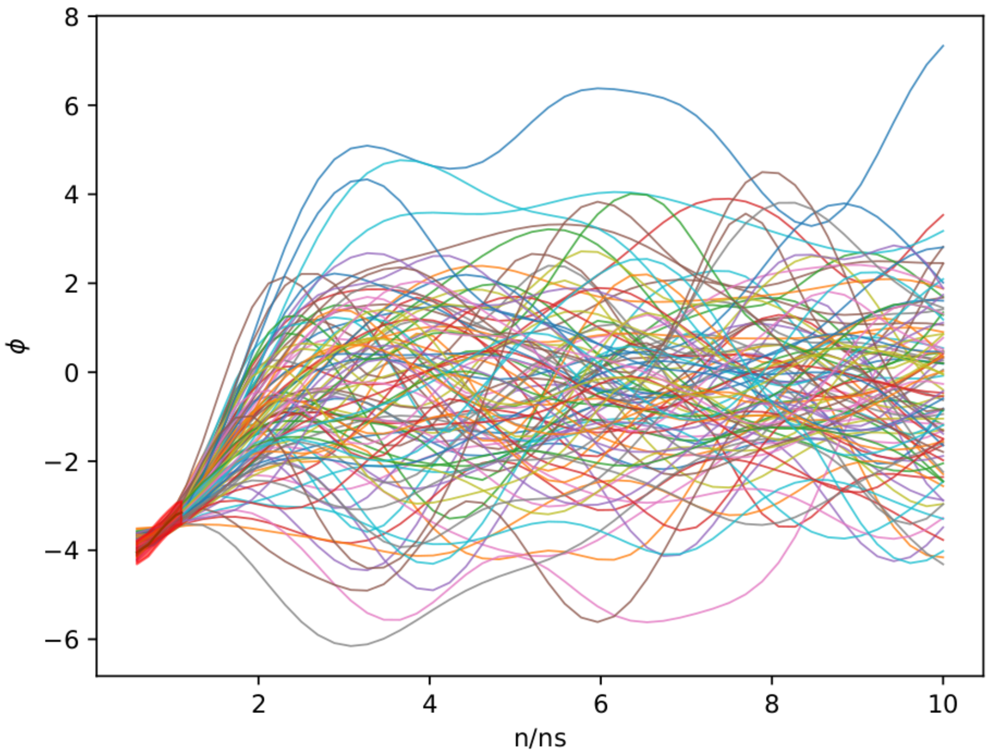

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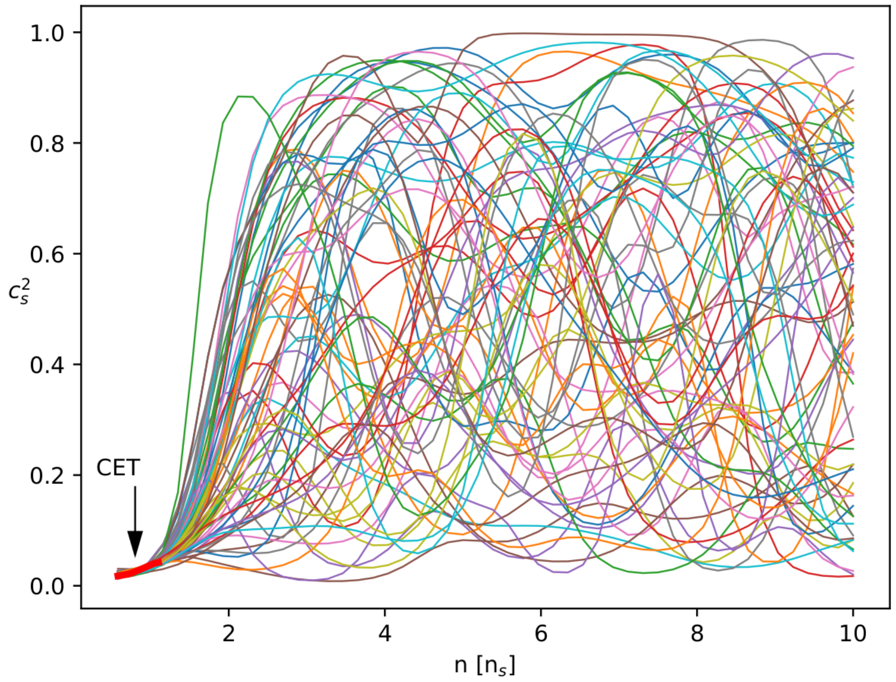

Setup

TG, Komoltsev, Kurkela, 2204.11877

1. Use **Gaussian-Process regression** in auxiliary variable $\varphi(n) = -\ln\left(c_s^{-2}(n) - 1\right)$ to extend CET EOS to $10n_s$

2. Fold in NS observations with full uncertainties
   - High-mass pulsars *(PSR J0348+0432 and PSR J1624-2230)*
     Approximate as Gaussians
   - GW170817
     Joint distribution on q and $\tilde{\Lambda}$
   - NICER measument *(PSR J0740+6620)*
     Joint distribution on M and R

3. Fold in QCD input as constraint at $10n_s$

Setup: Bit more about QCD constraint/likelihood

TG, Komoltsev, Kurkela, 2204.11877

1. Define triplet of thermodynamic properties:

$$\vec{\beta}_{\mathrm{QCD}}(X) = \{p_{\mathrm{QCD}}(\mu_H, X), n_{\mathrm{QCD}}(\mu_H, X), \mu_H\}, \quad X = \frac{3\bar{\Lambda}}{2\mu_H}$$

$X \in [1/2, 2]$ usually quantifies renormalization-scale dependence

Setup: Bit more about QCD constraint/likelihood

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$X \in [1/2, 2]$ usually quantifies renormalization-scale dependence

2. Create distribution on these properties at high density

$$P(\vec{\beta}_H) = \int d(\ln X) w(\log X) \delta^{(3)}(\vec{\beta}_H - \vec{\beta}_{\mathrm{QCD}}(X)), \quad w(\ln X) = 1_{[\ln(1/2),\, \ln(2)]}(\ln X)$$

suggested by Cacciari & Houdeau, JHEP 09, (2011)

Setup: Bit more about QCD constraint/likelihood

TG, Komoltsev, Kurkela, 2204.11877

1. Define triplet of thermodynamic properties:

$$\vec{\beta}_{\rm QCD}(X) = \{p_{\rm QCD}(\mu_H, X), n_{\rm QCD}(\mu_H, X), \mu_H\}, \quad X = \frac{3\bar{\Lambda}}{2\mu_H}$$

$X \in [1/2, 2]$ usually quantifies renormalization-scale dependence

2. Create distribution on these properties at high density

$$P(\vec{\beta}_H) = \int d(\ln X) w(\log X) \delta^{(3)}(\vec{\beta}_H - \vec{\beta}_{\rm QCD}(X)), \quad w(\ln X) = 1_{[\ln(1/2), \ln(2)]}(\ln X)$$

suggested by Cacciari & Houdeau, JHEP 09, (2011)

3. Komoltsev construction gives $\Delta p_{\rm min}$, $\Delta p_{\rm max}$ between $10 n_s$ and pQCD for each β_H:

$$P(\mathrm{QCD}\,|\,\mathrm{EoS}) = \int d\vec{\beta}_H P(\vec{\beta}_H) 1_{[\Delta p_{\rm min}, \Delta p_{\rm max}]}(\Delta p) = \int d(\ln X) w(\log X) 1_{[\Delta p_{\rm min}, \Delta p_{\rm max}]}(\Delta p)$$

Perform by substituting in $P(\beta_H)$, performing Monte-Carlo integration

Results

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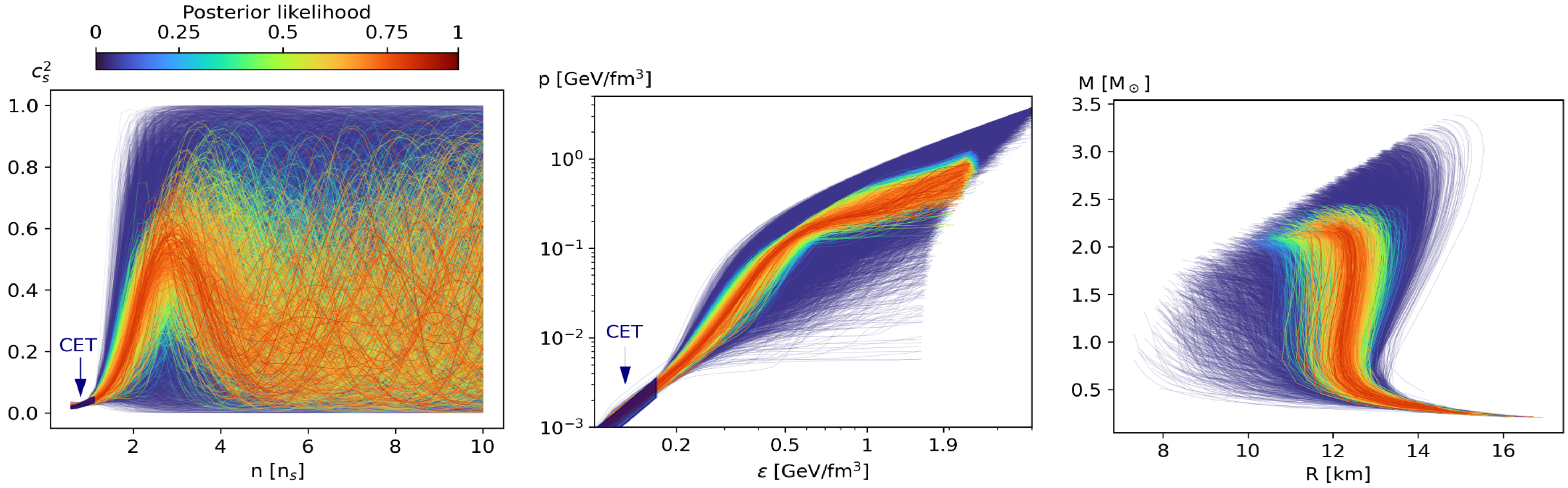

Results

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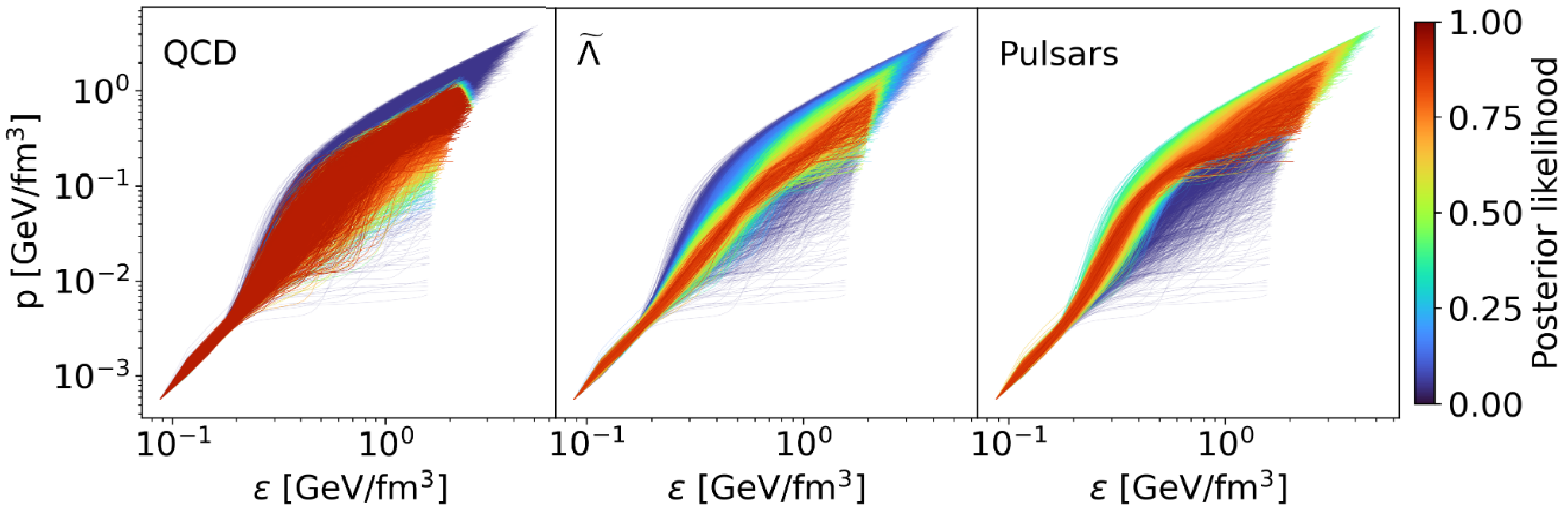

Results 1/2

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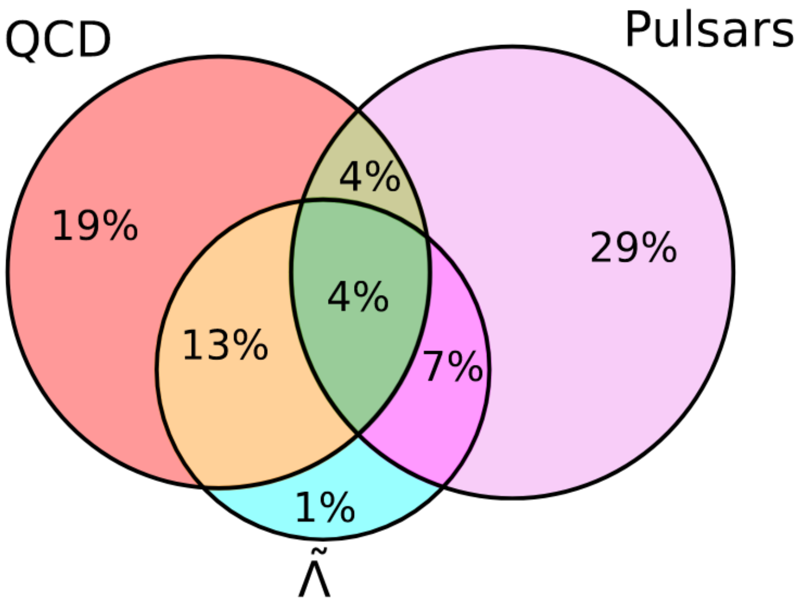
1. Inputs complementary



resample proportional to likelihood

Results 1/2

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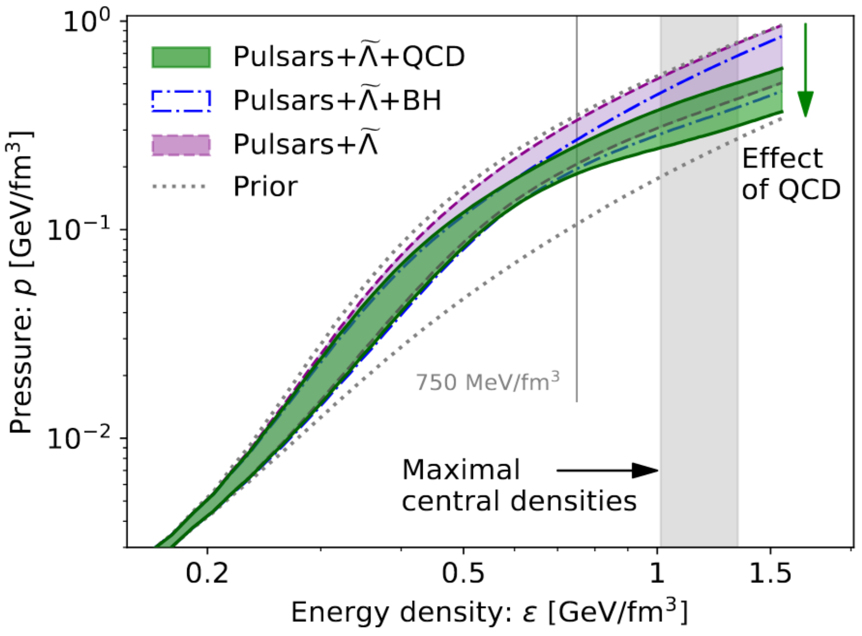
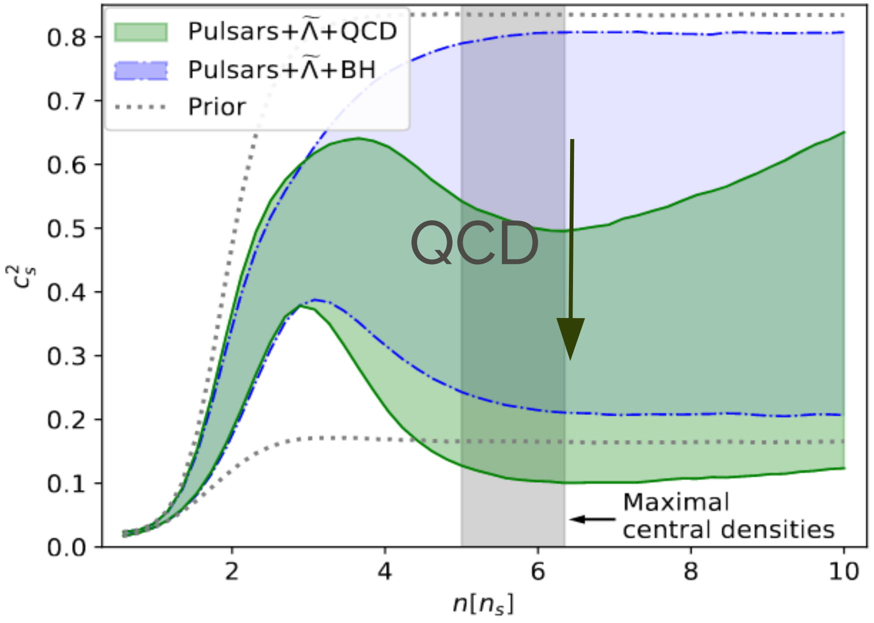
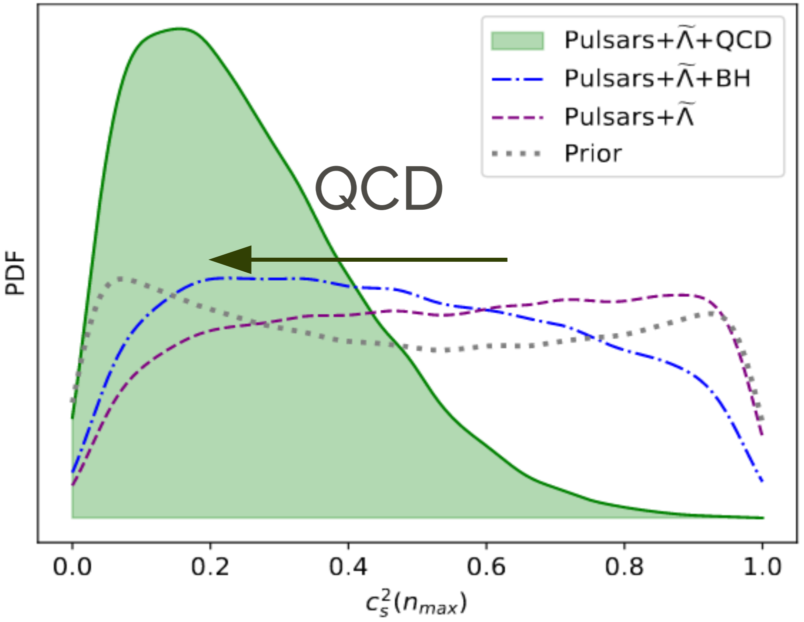
1. Inputs complementary

2. *QCD input softens the EOS*







Results 1/2

TG, Komoltsev, Kurkela, 2204.11877

1. Inputs complementary

2. *QCD input softens the EOS*

Key points:

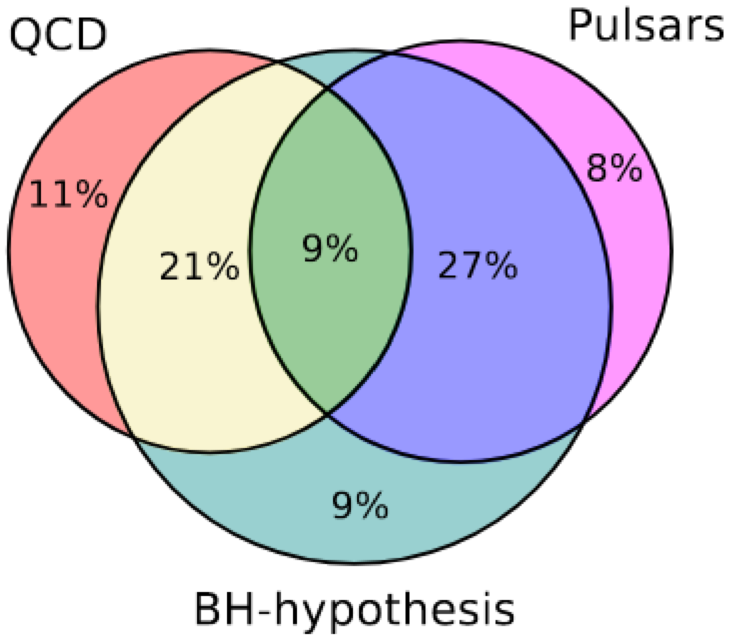
1. Overall, picture *consistent with hard-cut analysis*

2. *QCD impacts NS-EOS inference*

Results 2/2

TG, Komoltsev, Kurkela, 2204.11877

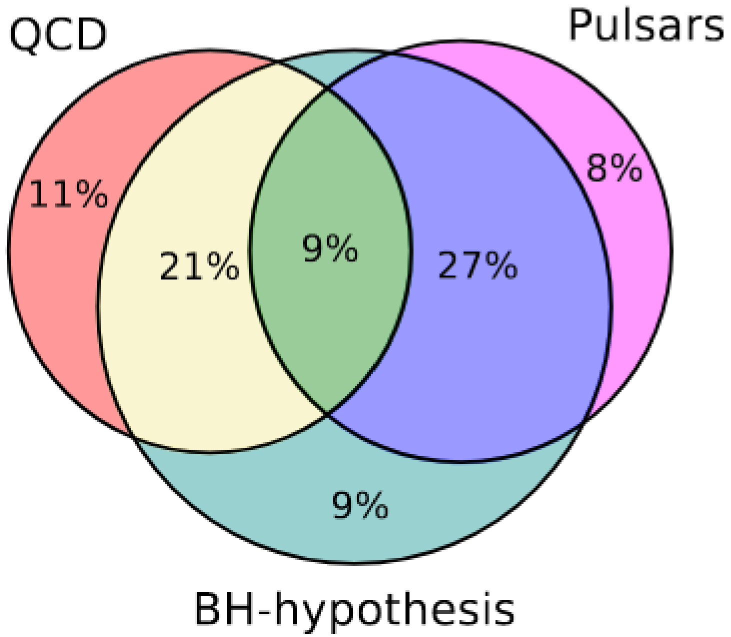
- Also see most overlap with BH-hyp (from GW170817). In fact QCD + astro → BH-hyp

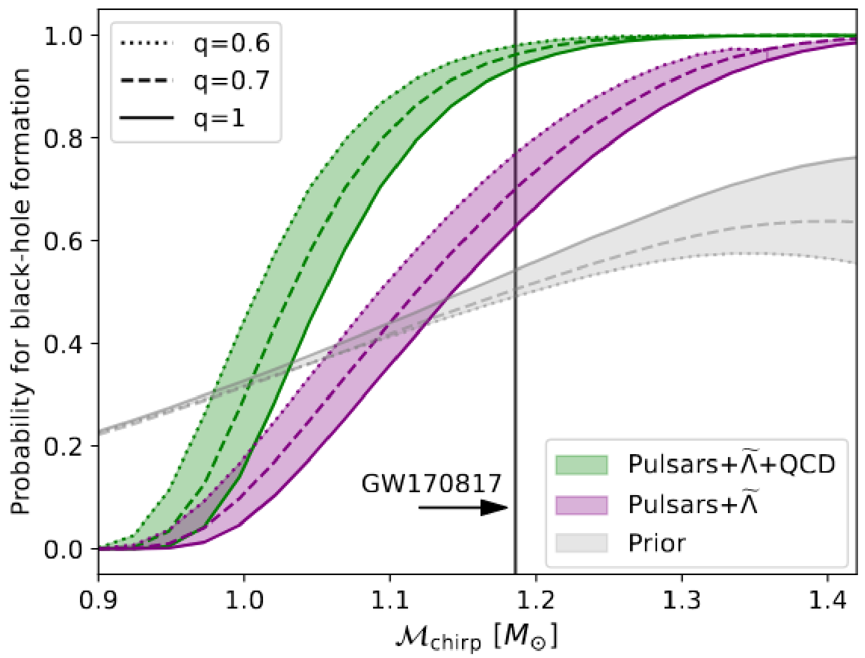


Results 2/2

TG, Komoltsev, Kurkela, 2204.11877

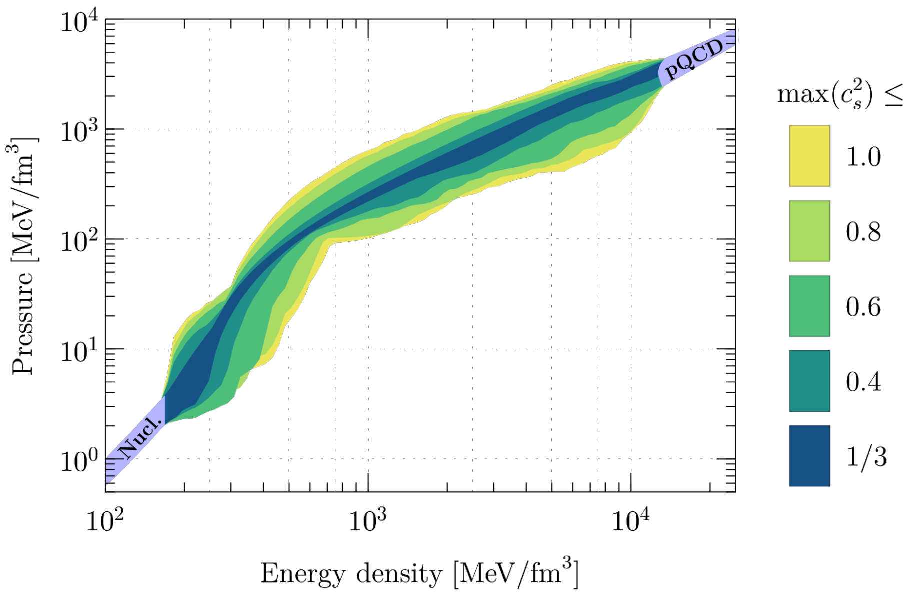
- Also see most overlap with BH-hyp (from GW170817). In fact QCD + astro → BH-hyp
- Also *generically predict* BH formation in most merger events





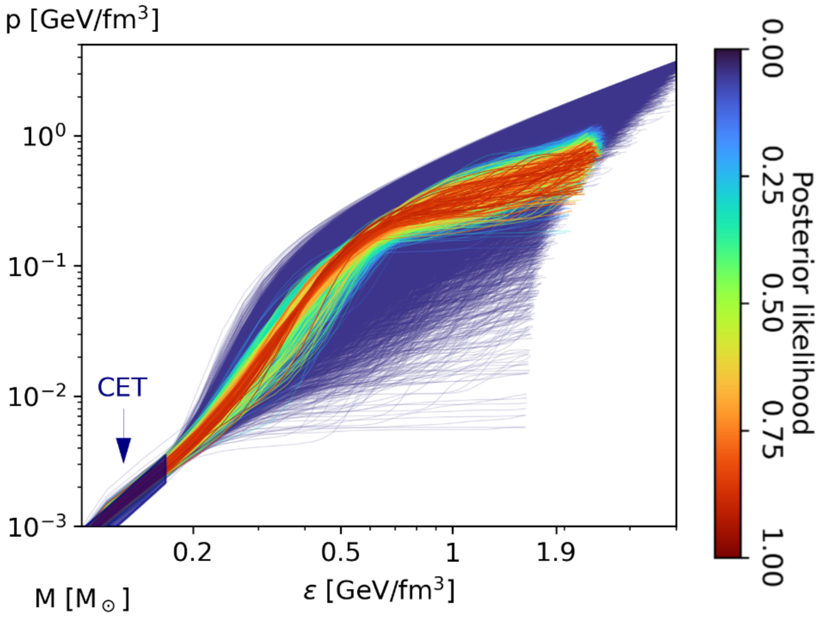
Takeaway: two approaches give similar results

1. Hard cuts



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

2. Likelihood analysis



TG, Komotsev, Kurkela, (To appear; 2204.XXXXX)

Takeaway: main conclusions from recent work

- *Should use QCD input in analysis of NS-EOS inference; it impacts the inference!*

  Jupyter notebook available on Github: OKomoltsev/QCD-likelihood-function

- QCD input at $10n_s$ *drives softening* in TOV stars / at high densities, as indicated in hard-cut analysis

- QCD input complementary to NS observational inputs

- See evidence for non-conformal → conformal transition, with thermodynamic properties transitioning from hadronic → quark
  - *Evidence for QM cores in massive NSs*